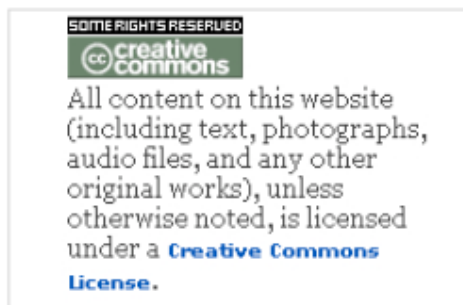


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# Emergence of representations from a multi-agent implementation of Schelling's model

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## Abstract

In this paper we use the concept of emergent representations issue from decomposition along two axes: collective/individual and internal/external. In a collective/internal composition, emergent representations are seen as internal, stable and non-reactive complex adaptive systems. Based on natural optimization algorithms, like Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO), we show how representations can emerge from Schelling's segregation model. Selection, building blocks and "inert" information requirements concerning emergence of representations are exemplified.

*Keywords: Swarm intelligence, cognitive psychology, multi-agents systems, emergent representations, Schelling's models.*

## 1. Introduction

We use an approach developed by Wilber (2000) to describe social-psychological systems and by Ferber (2007) to describe MAS (Multi-Agent Systems). We focus on decomposition into two dimensions: internal/external and individual/collective. Emergent representations are intermediate entities between a collective action of components (praxis of agents) and a holistic system composed by agents (Complex Adaptive System - CAS). In the field of CAS based on multi-agents, emergent representations could take place at the agent level as well as on organization level.

Rocha and Hordijk (2005) think emergent representations like genetic code. In artificial systems, Genetic Algorithms (GA) are combined to Automata Cellular (AC) to build representations. Symbolic based functions are spontaneously formed inside computing cells by selection and self-organization. Their objective was to make these functions able to keep a "long-term memory" of the "phenotype" of the CAS. Requirements for emergent "*material representations*" are proposed: "*dynamically incoherent memory*", "*construction code*" and "*self-organization and selection*" (Rocha and Hordijk 2005).

Steels (2003), Loula and Queiroz (2003) and Arnellos et al. (2006) present different MAS which negotiate symbols in a semiotic relationship ("Interpreting" = Agent, "Symbol" = Material Symbol and "Object" = virtual agents or objects). At the beginning agents do not have the same symbols referring to the same objects. Through interactions, the use of symbols to represent objects is mediated by reinforcement functions but depends on complex dynamics. A lexicon

takes the form of a CAS, where symbols represent external CAS objects. Representations in semiotic relationships are “*external emergent representations*” (Steels 2003).

Ramos and Almeida (2000) present a collective memory by using MAS. Agents mark their environment that consists in digital images. A pattern of these digital images emerges by color based marks. This system is completely reactive: when digital image change, the pattern changes too. An external observer can see an image-in-action because agents gradually change marking based on the colors. Carvalho and Hassas (2005) try to create “inert” dynamics like mental long-term representations through an agent-based model of collective sortius and Hope et al. (2006) try through a model to collective foraging.

We have proposed to understand research on emergent representations by using four kinds of models resulting from decomposition internal/external and individual/collective to describe different aspects of emergent representation models. Internal: stable and non-reactive informational values for guiding development. External: external representations concern external objects in semiotic relationships. Individual: emergent representation is material symbol systems. Collective: emergent representation is an emergent group of autonomous units of information processing (Carvalho et al., 2008).

A collective internal model can satisfy requirements for emergent representations. Dynamically incoherent memory needs building blocks. We think that an emergent group based on agents can play like memory blocks keeping “inert” an informational value. We propose to create emergent agent’s groups using Schelling’s model. Schelling’s model offers “*an arbitrary code*” able “*to construct arrangements of building blocks*”. Selection is viewed in terms of collective segregation, a threshold of tolerance to difference between agents. Rules of action inspired in social segregation in Schelling’s model implement emergent groups and organizations based on agents both self-organized.

Table 1: Four models of emergent representations. Analyze according to four quadrants resulting from the decomposition individual/collective and internal/external (Carvalho et al., 2008).

		Adaptive Complex systems (CAS)	
		Internal CAS	External CAS
Agent	Individual	<p><b>Individual internal</b></p> <ul style="list-style-type: none"> <li>- Change and exchange of symbolic functions that are inside each agent.</li> <li>- Representation is active in guidance towards an optimal form of organization.</li> </ul> <p>&lt;&lt; Genetic/biological cognitive systems and representations &gt;&gt;</p>	<p><b>Individual external</b></p> <ul style="list-style-type: none"> <li>- Negotiation and exchange of symbolic functions that are inside each agent.</li> <li>- Semiotic Representation (Agent, Symbol, Object outside the CAS).</li> </ul> <p>&lt;&lt; Emergence of language &gt;&gt;</p>
	Collective	<p><b>Collective internal</b></p> <ul style="list-style-type: none"> <li>- Interactions of agents and emergence of groups of agents.</li> <li>- Representation is active in the guidance towards an optimal form of total organization of agents.</li> </ul> <p>&lt;&lt; Cognitive activities of high-level, cognitive development, mental memory &gt;&gt;</p>	<p><b>Collective external</b></p> <ul style="list-style-type: none"> <li>- Interactions of agents and emergence of a global organization of agents.</li> <li>- Semiotic Representation (CAS, environmental marks, Pattern outside CAS).</li> </ul> <p>&lt;&lt; Social cognition, collective memory, social representations &gt;&gt;</p>

From dynamic approach in cognitive psychology, representations are internal systems inside complex and dynamic organisms (non reducible neither at the level of symbolic components of systems nor at the level of global system) (Gelder and Port 1995). Internal representations have been defined as physical symbol systems (Newell and Simon 1976). However, the concept of representation is often questioned. Even as biological organisms are very complex they do not need internal representations because the social and physical

environments are sufficient to guide this kind of system (Gibson 1979). In fact, mental high-level processes like cognitive development need dynamical representations (Gelder and Port 1995). Dynamical representations guide the intellectual development by inside and are essential in reasoning (Thelen and Smith 1994). Dynamic and self-organization is not sufficient concepts to understand cognition. Emergent representations are internal organizers (Steels 2003) guiding the system to optimal forms in a developmental way (Rocha and Hordijk 2005). Only self-organization does not ensure the best organization of MAS (Salima et al., 2006).

The objectives of this paper are: (i) present a collective internal model of emergent representations based on tolerance to difference inspired in human societies to solve optimization problems; and (ii) differentiate capabilities of this algorithm from others natural optimization algorithms in terms of emergence of dynamical memory of solutions.

Section *Schelling's segregation model* presents a background of Schelling's family models (Schelling 1971) and explains two existing implementations (Daudé and Langlois 2006). Next, in section *Dissatisfaction among minorities* we show that use little agent's groups to exploitation of design space is interesting. Next, section *From dynamical representations towards material emergent representations* proposes principles and an UML Meta-model of Emergent Representations (MER). In section *Adapting Schelling's model to optimization*, we describe how an Optimization by Tolerance to Difference (OTD) algorithm exploits and explores the design space in the optimization task. Section *An adaptive complex behavior without representations* shows a complex adaptive system based on Schelling's model able to optimization of different non-linear objective functions, like others natural inspired optimization methods (Kennedy and Eberhart 1995), (Dorigo 1996). Results aiming at illustrating the success of using the proposed methodology are reported in section *Emergent Representations: numerical experiments*. We show that is interesting use emergent groups based on agents to represent solutions of optimization problems. Performances in terms of size of error and stability over time are discussed on optimization of two functions with local/global optimums. Finally, concluding remarks and further research are presented.

## 2. Schelling's segregation model

In the 70's, Schelling studied how the cities can be structured in community blocks. Empirical investigations was not enough to understand the social phenomenon of segregation because they lead to the conclusion that people do not want to be majority in their communities, so it was difficult to explain segregation by prejudice. However, Schelling proposed a hypothesis enouncing that a small preference for one's neighbors to be of the same skin color could lead to total segregation. In other words, segregation is possible without individual will of segregation (Schelling 1971).

Beginning by a standard Schelling's model, we have given more attention to the asynchronous mode suggested by Daudé and Langlois (2006) than to the synchronous mode. In asynchronous mode, the procedure of simulation is:

1. Initialize a list of the inhabited "dwellings" (cells);
2. Initialize a list of all agents present in the MAS;
3. Repeat following procedure for each agent:
  - a. Random selection of an agent in the list,
  - b. The agent observes its neighborhood (eight neighbors),
  - c. According to its tolerance to difference, the agent remains or leaves.

The family of Schelling's models counts the following variables:  $N$  for the total number of cells,  $d$  for the global density of population counting one agent per cell,  $nC$  for the number of neighbor's cells,  $S$  for the tolerance to difference. The satisfaction of an agent depends on the number of foreign agents in his neighborhood and on his tolerance to difference. Hence, a general notation for Schelling's models is  $M(N, d, nC, S)$  (Daudé and Langlois 2006).

### 3. Dissatisfaction among minorities

We have implemented a model of Schelling with the following parameters:  $N = 2601$  cells,  $d = 98\%$  of each cell inhabited by just one agent;  $n = 3$  predefined groups (non-emergent) and  $S = 66\%$  concerning tolerance to difference. However, we have added one more variable on the standard model, the size of each group ( $t_n$ ). A simulation with three groups whose  $t_1 = 0.1 * d$ ,  $t_2 = 0.45 * d$  and  $t_3 = 0.45 * d$  is shown above. This Schelling's model is described by:  $M(N, d, n, S, t_n)$ .

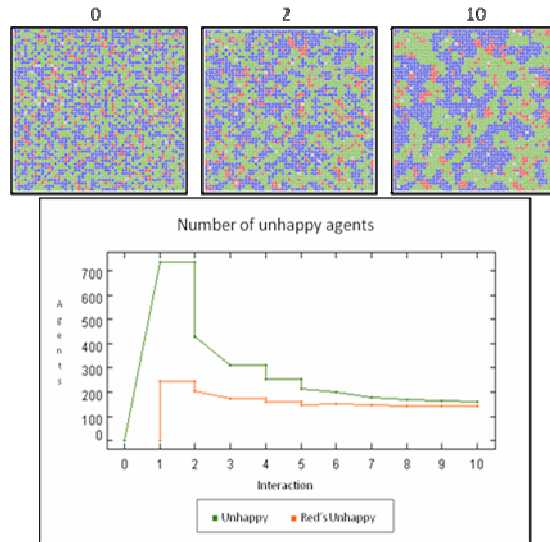


Figure 1: Among the three groups, the group of unsatisfied agents after 10 interactions is composed almost exclusively of agents  $w$  (reds agents).

According to their threshold of tolerance to difference, agents can be satisfied and remain in their cells. Dissatisfied agents move randomly to another cell. Agents dissatisfied does not converge because belong to the minority group.

Optimization methods have performances depending on their balance between exploitation and exploration of the solution space (Dorigo et al. 1996), (Kennedy and Eberhart 1995). A system with a good rate of exploitation keeps an informational value, i.e., a memory of the solution of a problem. However, if the system makes a lot of exploitation, it is possible it finds not better solutions than those it already found, i.e., there exist here a missing of exploration of the solution space. The best would be a system that does both things in certain equilibrium.

We have had the idea to use this kind of minority group to perform exploitation of solutions. The exploitation of the solutions of past problems is the main property of emergent representations because it acts like a memory.

### 4. From dynamical representation towards material emergent representation

We support the hypothesis that “construction code” (emergence of groups), “self-organization and selection” and “dynamically incoherent memory” are main concepts to define representations (Rocha and Hordijk 2005) because representations are not symbols but emergent and dynamical systems inside complex systems (Carvalho and Hassas 2005). Representations emerge on intermediate levels of a dynamical complex hierarchy like groups of autonomous units of information processing.

We propose a Meta-model of Emergent Representations (MER) with four principles:

1. Representations are emerging holistic groups in multi-agents complex adaptive system manifesting their existence through:

- a. A property belonging to an emergent group. This property is non reducible neither to any agent nor to the emergent global organization;
- b. Representations are embedded, embodied and open systems inside physical symbol systems. In the emergent group, agents are entering and leaving constantly the group.
2. Representation emerges embedded in a complex hierarchy as an intermediate system between the interaction of the agents (collective actions) and the global system (the CAS).
3. Inside CAS representation can play two roles: (i) to guide the CAS and (ii) to keep information dynamically "inert" (stable and non-reactive). Both roles and the fact that representation has a body characterize the "modes" of the existence of an emergent representation in an environment.

*mode (re, m, en)[role (re, r, cas)]* : the emergent representation *re* has a mode *m* in environment *en* [*re* has a role *r* in a complex adaptive system *cas*]

4. An emergent representation *re* can solve a problem. For optimization problems, emergent representation optimizes an objective function *oFunc* in a space of solutions *M*, if there is a mode *m* of *re* in *M*. In other words: in an environment *en* an adaptive action *o* has rules of action in the agent's architecture *a1, ..., an* and makes part of a *m*. As a result:

*en: m.o (a1, ..., an) → re : EmergentRepresentation, mode (re, m, en) type (m, M)*  
*oFunc(o(a1, ..., an), M)*

Any agent is a member of (at least) one group:  $\forall x: Agent, g:Group, member(x, g)$

- a. The body of an emergent representation, i.e., an emergence of group based on agents, emerges through rules of action implemented in agents' architecture;

$\forall x: Agent, g:Group, member(x, g)$

- b. An adaptive action *o* (based on rules of actions *a1, ..., an* in the architecture of agents), led to the emergence of an open group of agents:

$\forall re: EmergentRepresentation, \forall cas: ComplexAdaptifSystem, \forall r: Role\ plays(re, r, cas) \rightarrow gs: GroupStructure\ GStruct(g, GS)\ o(g, GS)$

The UML (Unified Modeling Language) diagram concerning MER, a Meta-model of Emergent Representations is proposed.

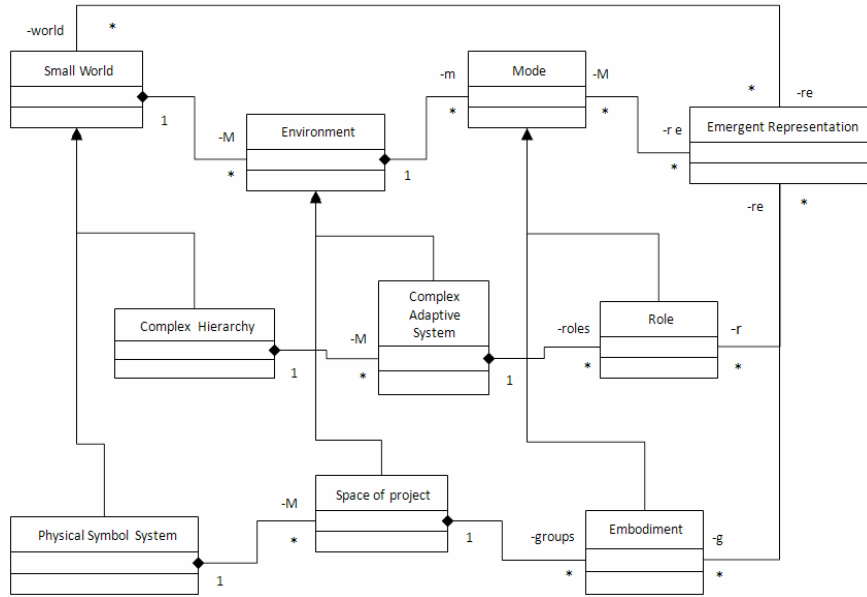


Figure 2: The UML diagram of the MER.

Emergent Representations (RE), implemented in a low-dimensionality, have modes of action on environmental problems. RE's modes are its roles and its bodies. Representations have two roles: (i) adapt or solve a problem and keep information on the solution space stable (ii) guide the development of CAS. The body configuration is done through the emergence of an open group of agents that are embedded in the space of solutions. In the case of optimization problems, the aim is to minimize or maximize the solution. An "inert" emergent group inside CAS has its own property: this group exploits the optimal solution of a function  $oFunc$ .

## 5. Adapting Schelling's model to optimization

In this section, we consider functions of type  $y=f(x)$ . Each agent has information about the value  $y$  at a given time and the position of the group is supposed to give an estimation of the corresponding value  $x$  (the solution). The smallest group is called *group A* and agents belonging to this group are denoted  $w$ . At the beginning, agents have a low tolerance ( $S = 14\%$ ). Agents  $w$  increase their tolerances in reaction to the  $y$  values of the objective functions, randomly encountered in their neighborhoods.

$$Tolerance(t) = Tolerance + Amplifier * \mu \quad (1)$$

The variable  $\mu$  corresponds to an average of local values of  $y$  in the neighborhood of an agent  $w$ . Each agent  $w$  observes in its neighborhood the existence of points  $P$  belonging to an objective function  $oFunc$ .

Below we see the convergence among the most satisfied agents  $w$ , i.e., an emergent group. As a real social actor does not exploit only the best source at his disposal, we chose to use averages rather than adopting the *pbesty* like Kennedy and Eberhart (1995). In the implementation, agents  $w$  are represented by the color red. Red agents belong to *group A*. In some cases, values of  $\mu$  are so small that agents remain, despite any differences they can meet in their neighborhood.

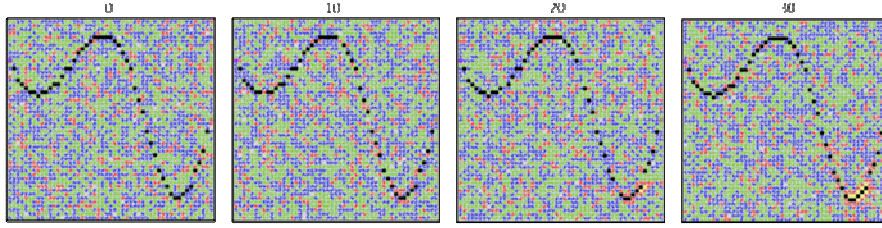


Figure 3: Optimization of a discrete and nonlinear function  $aFunc1$  with an initial tolerance level to 14% and with 3 groups. Agents  $w$  (red at the beginning), once satisfied (6 or more out of 8 neighbors of type  $w$ ) become yellow.

Yellow agents belong to *group A*. The group of yellow agents is an emergent group, called *group B*. The *group B* is a self-organized building block embodied like an open system in this CAS. Selection is made by segregation and its informational value is stable. But is this informational value reactive or a “dynamically incoherent memory”? These results led us to look for the change of the objective function during the execution of the algorithm in order to evaluate complex adaptive system.

## 6. An adaptive complex behavior without representations

We have experimented change the objective function. We have added the possibility to change the objective function, from  $oFunc1$  to  $oFunc2$ , during execution. Our observations are the following:

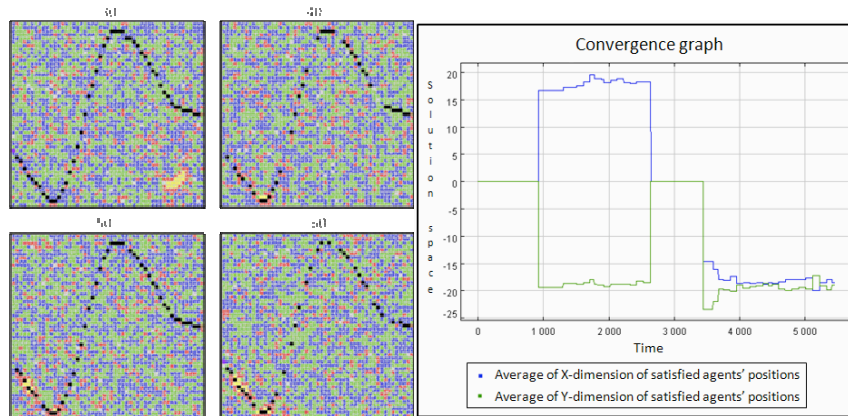


Figure 4: Adaptation of the system during the execution of the algorithm while changing the objective function  $oFunc1$  to  $oFunc2$ . It realizes adaptation without representations.

The convergence graph above shows a stability of information by an average of the values of XY-plan. Function  $oFunc1$  is replaced by  $oFunc2$  at time 2600.

In a collective internal point of view of emergent representations a collective memory such as an emergent group that keeps stable the informational value concerning  $oFunc1$  is essential. While it is not observed, the system is still reactive. This is the case in the system described above.

## 7. Emergent representations: numerical experiments

Second order cybernetics says that elements change behavior when they perceive emergent properties of the system that they are part of. To make possible the reaction of agents  $w$  to the emergence of *group B*, agents  $w$  decide if they stay or if they move based on a threshold of tolerance which depends on: the initial tolerance, the value of  $\mu$  and the number of



agents yellow  $J$  in their neighborhoods. The final threshold of tolerance is inversely proportional to  $\mu$ , however, directly proportional to  $J$ . Below, the emergent yellow *group B* keeps a representation of solution of *oFunc1*. *Group A* explores yet solution space and optimize *oFunc2*.

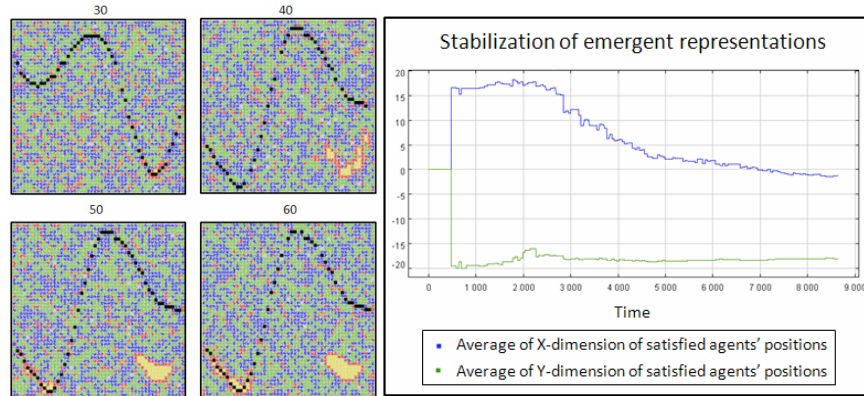


Figure 5: Emergent representation: a building block based on emergent group, selection by social segregation, informational value stable and non-reactive. Graph shows a time-varying stabilization analysis of emergent representations. The objective function *oFunc1* is replaced for the objective *oFunc2* in time 2000 i.e. iteration number 40.

The main requirement of emergent representations in a collective internal model is fulfilled. “Material representations demand inert physical structures [...]. The role of these physical structures is not defined by their dynamic characteristics but rather by their informational value.” (Rocha and Hordijk 2005).

The balance between exploration and exploitation exists in *group A*. *Group A* has an exploitation property because it is the minority group. Inside *group A* emerges *group B* that has an exploitation property because it keeps stable its informational value. *Group B* has a dynamically incoherent memory role in this CAS based in Schelling’s model.

## 8. Conclusion

Considering four kinds of models of emergent representations, we have proposed the decomposition internal/external and individual/collective to describe different aspects of emergent representation models. Each component underlines certain aspects. Internal: stable and non-reactive informational values for guiding development. External: external representations concern objects in semiotic relationships. Individual: emergent representation is physical symbol systems. Collective: emergent representation is an emergent group of autonomous units of information processing.

We have shown how a collective internal model can satisfy requirements for emergent representations: dynamically incoherent memory, construction code, self-organization and selection.

Emergent representations, studied by numerical experiences, are particularly interesting in psychology and cognitive sciences because it refers to material or physical dynamical nature of intelligence and cognition. Cognitive systems are viewed as complex adaptive systems supporting emergent representations. Our theoretical approaches concern the dynamical and distributed paradigm on computer science and cognitive sciences, i.e., the parallel/distributed computing in artificial intelligence and artificial life. We have chosen a reactive approach in multi-agents paradigm (Hassas et al. 2006). This approach is based on emergence that replaces a centralized control, thus making the system simple instead of complicated.

Further researches consider artificial applications of the algorithm for driving learning on neuronal networks, in particular on the training of logical function XOR and consider tests with different functions related variables:  $T$ ,  $\mu$ ,  $J$ . The standardization of an optimization method

based on the Optimization based on the Tolerance to Difference (OTD) is also envisaged. Concrete applications are envisaged to model market micro stabilities or equilibriums in a game theory perspective (local Nash equilibrium).

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