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APOCOSIS

Population growth and social welfare: a dynamic model approach

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Abstract

We present in this paper a deterministic model for a general human population dynamics. The main variables are the population and the welfare variables. These variables are considered in the IDH [1] calculation. The model has been validated for the case of Spain in the period 1996-2005.

Keywords: Human Development Index (IDH), dynamics, population density, welfare variables, fertility rates, death rates, immigration rates, emigration rates.

Introduction

After working with population dynamics models, our interest is focused now on the introduction of welfare variables.

In this work we show a first approximation, and therefore, we do not distinguish between sexes or ages; we consider only total populations.

Trying to carry out a rigorous work we have checked in the UNO publications [1] about the Human Development Index (IDH).

The IDH is about of a compound indicator which measures the average advance of a country depending on three basic dimensions of the human development:

- Long and healthy life (life expectancy at birth).
- Access to knowledge (adults' literacy rate and matriculated-persons' gross-rate in primary, secondary and tertiary education).
- Appropriate level of life (interior gross product).

These basic dimensions are measured up according to the variables that we have indicated in brackets.

Although the concept of human development is wider enough to be measured by only one index, the IDH is the alternative to the use of the Interior Gross Product (PBI) [1], as a measure of the human well-being and it is useful to approach the information located in the tables of UNO indicators, which we can find in [1].

We use the following four basic welfare variables in our model:

- PBI Interior Gross Product
- EV Life expectancy at birth.
- BM Registered school population.
- AA Adult Alphabetized Population.

[1] Human Development Index, calculated by the UNO.

The definition of these four welfare variables allows us to create a deterministic model through the following steps:

- Find the relevant factors variables.
- Find the influence relationships between the factors or variables previously identified.
- Find the equations or functional relationships that allow determining the behaviour of each variable.
- Search for the necessary historical data for the calculation of the numeric constants that appear in the global model of the behaviour of the system.
- Validate the model.

Variables identification

In this work, we have followed the Brainstorming method [8] to identify the elements. The codification consists of giving a name to each element taken into account.

Input variables for IDH calculation and its codification:

- EV (t) Life expectancy at birth.
- PBI(t) Interior gross product per capita.
- BM(t) Registered school population.
- AA(t) Adult alphabetized population.

Input variables for population calculation and its codification:

- NAC(t) Births population.
- DEF(t) Deaths population.
- INM(t) Immigrant population.
- EMI(t) Emigrant population.

For our model we try to obtain the birthrate, death rate, immigration rate and emigration rate, depending on the IDH. Therefore the variables are: TNAC (IDH), TDEF (IDH), TINM (IDH) and TEMI(IDH) respectively.

Output variables to calculate the population and its codification.

- POBT(t) Total population.

Variables relationing

To see the how some variables influence on others, we present the Forrester's graph (Annex I) corresponding to our model.

Calculating IDH

As we have said in the Introduction, the IDH is a compound indicator. To calculate IDH we use maximum and minimal values [1], that we give in Table 1 of Annex I.

Equation 1: Life Expectancy Index.

$$IEV(t) := \frac{EV(t) - \min ev}{\max ev - \min ev} \qquad (1)$$

Equation 2: Literacy Rate Adults [2].

$$TAA(t) := \frac{AA(t) - \min aa}{\max aa - \min aa} \qquad (2)$$

Equation 3: Gross Rate Registered to level primary, secondary and tertiary[3].

[2] AA(t) have been obtained dividing Alphabetized Adult Population and school population(age period 0-16).

[3] BM(t) have been obtained dividing Registered School Population and school population(age period 0-16).

$$TBM(t) := \frac{BM(t) - \min bm}{\max bm - \min bm} \qquad (3)$$

Equation 4: Educational Index.

$$IED(t) := \frac{2}{3} TAA(t) + \frac{1}{3} TBM(t) \qquad (4)$$

Equation 5: Interior Gross Product Index.

$$IPIB(t) := \frac{PIB(t) - \min pib}{\max pib - \min pib} \qquad (5)$$

Equation 6: Human Development Index.

$$IDH(t) := \frac{1}{3} IEV(t) + \frac{1}{3} IPIB(t) + \frac{1}{3} IED(t) \qquad (6)$$

Fitting the input variables

We have obtained the real data population from the Spanish National Institute of Statistics database [2]. In particular, we have obtained information in the period 1996-2005 corresponding to the following variables:

- Life expectancy at birth.
- Interior Gross Product per capita.
- Registered School Population.
- Alphabetized Adult Population.
- School population.
- Births.
- Deaths.
- Immigrant.
- Emigrant.
- Total Population

The fitted functions have been obtained by using the packet *NonlinearRegression* from MATHEMATICA 6.0.

We comment next how the input variables have adjusted from this information and how the possible faults of information have been resolved.

The justification of the choice of these adjustments is given through the figures shown in Annex I.

The life expectancy index has been calculated from Equation (1) with life expectancy information in the period 1975- 2005, and we have fitted it with the sum of two logistics.

$$IEV(t) = \frac{\alpha_1}{1 + \beta_1 \cdot e^{\delta_1 \cdot (1975 - t)}} + \frac{\alpha_2}{1 + \beta_2 \cdot e^{\delta_2 \cdot (1993 - t)}} .$$

The interior gross product index has been obtained from Equation (5) with information in the period 1996- 2006, and it has been fitted with a logistics.

$$IPIB(t) = \frac{\alpha_3}{1 + \beta_3 \cdot e^{\delta_3 \cdot (1996 - t)}} .$$

The adult literacy rate has been obtained from Equation (2) with information in the period 1991-2005 and it has been fitted with a logistics.

$$TAA(t) = \frac{\alpha_4}{1 + \beta_4 \cdot e^{\delta_4 \cdot (1991 - t)}} .$$

The matriculated persons' gross rate has been obtained from Equation (3) with information in the period 1998- 2006 and it has been fitted with a logistics.

$$TBM(t) = \frac{\alpha_5}{1 + \beta_5 \cdot e^{\delta_5 \cdot (1998 - t)}} .$$

According to the UNO, the educational index is defined as

$$IED(t) := \frac{2}{3} TAA(t) + \frac{1}{3} TBM(t) ,$$

So in our case it is given with the sum of two logistics.

Wit respect to the IDH, it will also be a sum of logistics, as we can see through Equation (6).

In Figure 6 of Annex I we can observe that the function IDH is a very good approximation of the real information obtained by the UNO.

Now we comment about the adjustment of the birthrate, mortality rate emigration rate and immigration rate. Here we present an innovation with respect to population dynamics models [3], [4], [5], [6], [7]. The functions on which the population depends are calculated by us depending on the IDH, a welfare variable, and not depending on time.

Specific information is shown in Annex I. The used information is the IDH corresponding to the years 1998-2005. Also in Annex I, all the graphs showing the real data and the corresponding approximate data obtained from these functions are presented.

With respect to the birthrates corresponding to the application period (1975-2007), they have been obtained dividing births and population. To fit the birthrate as a function of the IDH, we have also considered a logistic curve.

$$TNAC(idh) = \frac{\alpha_6}{1 + \beta_6 \cdot e^{\delta_6 \cdot (0{,}874714 - idh)}} .$$

This function is positive and satisfies the topological property of the birthrate (if the IDH increases then the birthrate too).

The death rates corresponding to the application period (1996-2006) have been obtained dividing deaths and population. To fit the death rate as a function of the IDH, we have also considered a logistic curve again.

$$TDEF(idh) = \frac{\alpha_7}{1 + \beta_7 \cdot e^{\delta_7 \cdot (0{,}874714 - idh)}} .$$

This function is decreasing and satisfies property that a higher IDH (more quality of life) implies a smaller death rate.

The emigration rates corresponding to the application period (1998-2004) have been obtained dividing emigrations and population. The function considered to fit this variable as a function of the IDH has been a logistic curve, as is shown in [7].

$$TEMI(idh) = \frac{\alpha_8}{1 + \beta_8 \cdot e^{\delta_8 \cdot (0{,}874714 - idh)}} .$$

This function increases until it remains constant at a certain IDH value and so it shows the same property that emigration.

The immigration rates corresponding to the application period (1998-2004) have been obtained dividing immigrations and population. The function considered for fitting has been also a logistic curve, as it appears in [7].

$$TINM(idh) = \frac{\alpha_9}{1 + \beta_9 \cdot e^{\delta_9 \cdot (0{,}874714 - idh)}} .$$

This function also increases until it remains constant at a certain IDH value and so it shows the same property that immigration.

The mathematical structures of the fitted functions corresponding to the emigration rate and immigration rate are also the indicated by [4] for time rates, i.e., logistic functions (or sums of them).

Model Equations

Taking into account all the input variables, the applied model to be validated is the following one:

Equation 7: Total population

```
t0=1996;
dt=1;
p0=POBT(t0);
d= Dimension of the data population-table.
for( t=t0+dt; POBT=p0; T={{t0,p0}}, t≤t0+d-dt, t=t+dt,
    IED=(2/3)·IAA(t)+(1/3)·IBM(t);
    IDH=(1/3)·(IEV(t)+IPBI(t)+IED(t));
    n=TNAC(IDH);
    df=TDEF(IDH);
    i=TINM(IDH);
    e=TEMI(IDH);
    nac=n·p;
    def=df·p;
    inmi=i·p;
    emi=e·p;
    POBT=POBT+dt·(nac-def+inmi-emi);
; T=Append[T,{t,POBT}]]
```

Model validation

To validate the presented model, we have written it as a set of finite difference equations (already showed previously). These equations have been programmed in MATHEMATICA language by introducing the functions fitted in the section "Fitting the input variables". Results have been obtained for the period 1996-2005, and the validation has been realized graphically superposing the results obtained for every year and the historical data.

The validation has been considered successful due to three reasons:

- The graphs are superposed in the first years in an almost exact form.
- We have calculated the determination coefficients (an index of global validation between data sets):

$$R^2 = \frac{\left(\sum_i (x_i - \mu_x)(y_i - \mu_y)\right)^2}{\sum_i (x_i - \mu_x)^2 \sum_i (y_i - \mu_y)^2},$$

where (x_i, y_i) are the data to be compared, and μ_x and μ_y are the respective average values.

- We have verified the randomness of the results by means of the maximum relative error.

To check this information, see Figure 11 in Annex I.

Conclusion

The presented model is a substantial advance with respect to the rest of population dynamics models, due to the fact that we have introduced the welfare variables considered by the UNO.

Any model in [4], [5], [6], [7] or [10] considers the population affected by the birthrate, death rate, immigration rate or emigration rate. We have managed a way to introduce the welfare variables mentioned by UNO by using the IDH as a determining factor for the rates above mentioned.

With respect to validation, we can say that, in spite of accepting the performed validation process from a theoretical point of view, we see graphically that from year 2000, the fitting curve separates considerably from the data. This leads us to think that we must introduce in the model the distinction by ages in order to improve the adjustment of the death rate and the emigration rate. That will probably improve the adjustment of population curves to historical data.

A challenge for future research can be the distinction per sexes and ages, since all our input variables not only depend on time and, due to other models [6], [7], [9], we know that the model validations are improved by introducing distinction per sexes and ages.

Another approach could be obtained by the conversion of this deterministic model into a stochastic model [3].

Finally, we also think about elaborating a generic model, which could be used for any country already developed or in developing process, in order to look for solutions to the sustainable growth of the societies in which the generational relief is in danger.

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ANNEX I

These are Forrester's graph, the tables and graphs that we have used in order to obtain the approximation of the output variables and the model validation.

In the graphs, lines indicate approximate information and points indicate the real data obtained from the data bases (I.N.E).

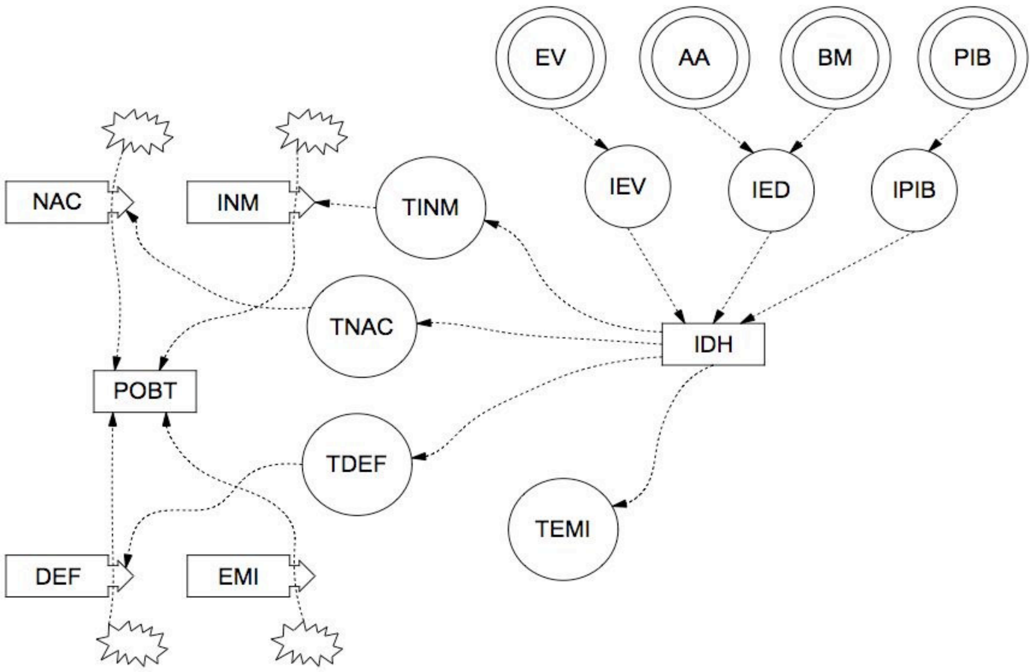

Figure1: Forrester's graph.

	EV (años)	PBI ($)	AA (%)	BM (%)
max	85	40000	100	100
min	25	100	0	0

Table 1. Maximum and Minimum Indexes provided by the UNO for the calculation of the IDH.

α_1	α_2	β_1	β_2	δ_1	δ_2
0.86081	0.0519079	0.0677235	5.11512	0.162474	0.292326

Table 2. Parameter Values for IEV(t).

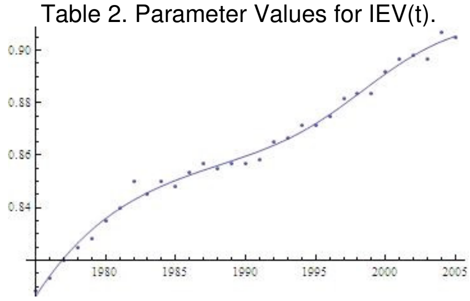

Figure 2: Fitted function (solid line) and real data (dots) for Life Expectancy Index in Spain, in the period 1975- 2006, R^2 = 0.992224. For Kolmogorov-Smirnov's contrast, the theoretical deviation D(α,17)> 0.0775546, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_3	β_3	δ_3
1.21	0.400867	0.0455412

Table 3. Parameter Values for IPBI(t).

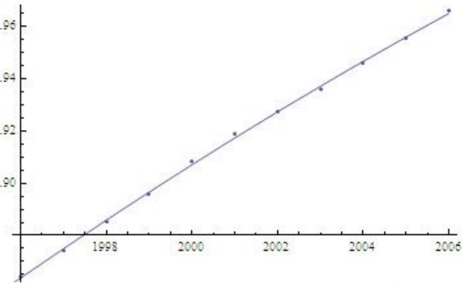

Figure 3: Fitted function (solid line) and real data (dots) for Interior Gross Product Index in Spain, in the period 1996-2006, R^2 = 0.999091. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 17) >$ 0.257384, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_4	β_4	δ_4
1.83226×10^{11}	2.29221×10^{11}	0.00678878

Table 4. Parameter Values for TAA(t).

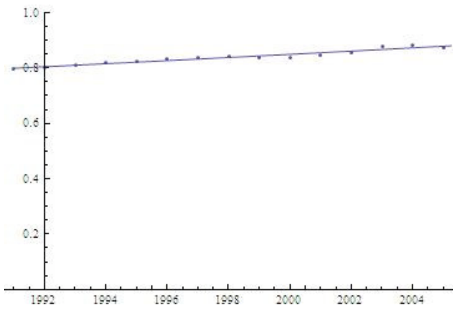

Figure 4: Fitted function (solid line) and real data (dots) for Literacy Adults Rate in Spain, in the period 1991-2005, R^2 = 0.933121. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 17) >$0.101194, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_5	β_5	δ_5
0.9723	0.106928	0.864894

Table 5. Parameter Values for TBM(t).

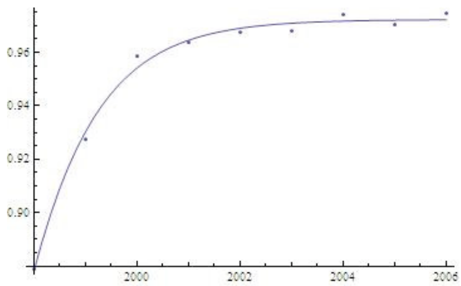

Figure 5: Fitted function (solid line) and real data (dots) for Registered School Population Rate in Spain, in the period 1998-2006, R^2 = 0.993383. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 17) >$)> 0.200483, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

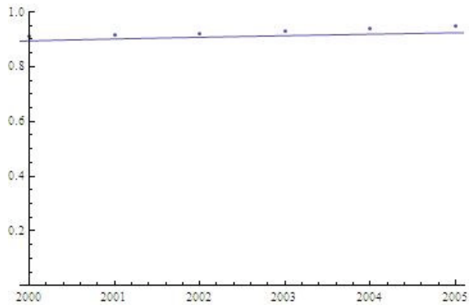

Figure 6: Fitted function (solid line) and real data (dots) for Human Development Index in Spain, in the period 2000-2005, R^2 = 0.933947. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 17) >$ 0.219809, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_6	β_6	δ_6
1.46722×10^{-7}	-0.999984	-0.0000451851

Table 6. Parameter Values for TNAC(IDH).

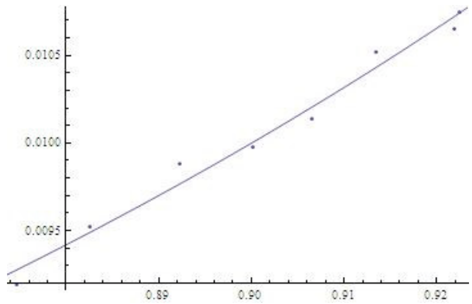

Figure 7: Fitted function (solid line) and real data (dots) for Birthrate depending on the IDH in Spain, from the IDH in the period 1998-2005, R^2 = 0.981883. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 17) >$ 0.173592, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_7	β_7	δ_7
0.00794512	-0.129136	7.4472

Table 7. Parameter Values for TDEF(IDH).

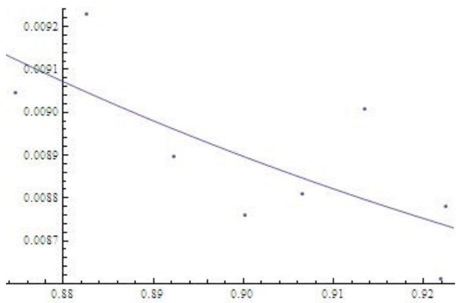

Figure 8: Fitted function (solid line) and real data (dots) for Death Rate depending on the IDH in Spain, from the IDH in the period 1998-2005, R^2 = 0.524619. For Kolmogorov-Smirnov's contrast, the theoretical deviation $D(\alpha, 17) >$ 0.231418, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_8	β_8	δ_8
0.00043113	8.24274	115.009

Table 8 Parameter Values for TEMI(IDH).

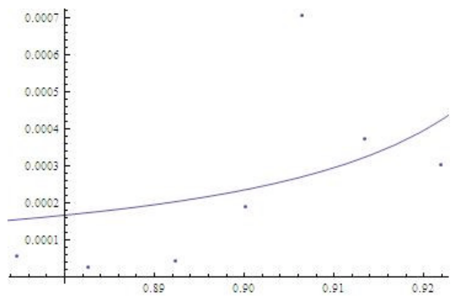

Figure 9: Fitted function (solid line) and real data (dots) for Emigration Rate depending on the IDH in Spain, from the IDH in the period 1998-2005, R^2=0.286699. For Kolmogorov-Smirnov's contrast, the theoretical deviation D(α, 17)>0.213082, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

α_9	β_9	δ_9
0.0137708	6.65123	103.68

Table 9. Parameter Values for TINM(IDH).

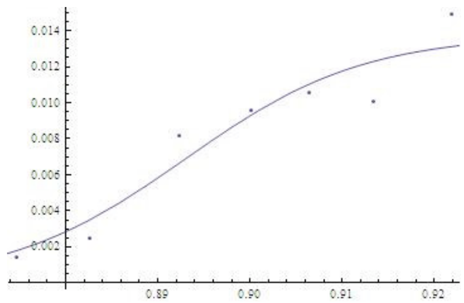

Figure 10: Fitted function (solid line) and real data (dots) for Inmigration Rate depending on the IDH in Spain, from the IDH in the period 1998- 2005, R^2 = 0.911174. For Kolmogorov-Smirnov's contrast, the theoretical deviation D(α, 17)> 0.155051, for any level of significance $\alpha \geq 0.01$. So, the normality hypothesis is accepted for the residues of the model.

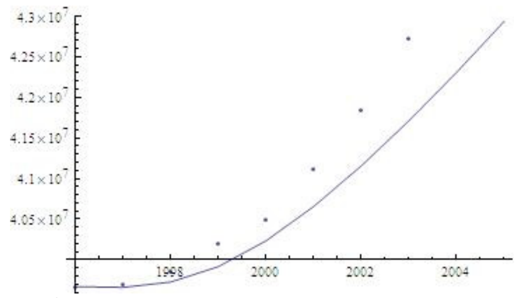

Figure 11: Forecast function (solid line) given by Model Equation and real data (dots) for Spain Population, in the period 1996-2005, R^2 = 0.878719. With 2.65633% of maximum relative error. The model is considered validated, since it does not overcome 5 %.