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Systems as moving and grouped manifolds. Metric transition term meaning and curvature impact.

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Systems as moving and grouped manifolds. Metric transition term meaning and curvature impact.

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Abstract

Volumes described by primitive systems (in the Kron's meaning[8]) when their currents take all possible values, create manifolds. For various environment conditions, parameters values change with time. It means that previous manifolds change in forms when the environment changes. To make a system, many of these manifolds are used, glued each other. So, when environment changes, the all structure is deformed. The purpose of this paper is to give some track to translate mathematically these deformations and to show how the metric incorporates the transition states and the meaning of these terms. Finally, we take a look on the curvature impact of these impedance definitions. This discussion follows previous ones presented in [9][10][11].

1 Introduction

Even if it's rarely specified, any impedance operator acts on given domains[1]. For example a resistance follows the law $U = RI$ only on a given domain of temperature and current amplitude. If the temperature is too high, the law can become $U = R_0 (1 + \alpha T)$. Logistic functions are used to define the domain and attach each domain a various law for one impedance operator. By the fact, the system of equations coming from these impedances and that describes the system can change of equations depending on environment parameters. As a manifold can be associated with each system of equations and its domains, this mechanism leads to moving manifolds[2]. But a system is made of various of these manifolds. Finally the whole assembling of manifolds moves when its environment changes.

2 Domains function

First step is to dispose of functions in order to be able to write, for any operator z[3]:

$$z_{ij} = \sum_n \mathcal{D}_n z_{ij}(n) \tag{2.1}$$

this for one parameter. For m parameters, the relation becomes:

$$z_{ij} = \sum_n \Pi_m \mathcal{D}_n^m z_{ij}(n) \tag{2.2}$$

A kind of function that answer well to the requirements are the logistic functions[4]. For example, to define on a domain $[a, b]$ some parameter q driven impedance, we can write:

$$\left(1 + e^{-\alpha(q-a)}\right)^{-1} - \left(1 + e^{-\alpha(q-b)}\right)^{-1} \tag{2.3}$$

Playing on delays and α, transition speed between two laws can be set as desired, as the domain width. Previous expression allows to obtain relations like (2.1) or (2.2).

3 Primitive manifold changing

We consider an impedance defined with domains. This definition is linked with a curve giving for all possible values or one variable x^k the result e_m through $e_m = \sum_n \Pi_m \mathcal{D}_n^m z_{mk}(n) x^k$, and this for one set of values for the parameters (x^k begin a generalized flux - i.e. currents in the electrical case). If the parameter values change, the impedance function changes and the previous curve becomes a set of curve, i.e., a surface. This surface can be seen as a manifold issued from a moving curve. As an example, we make a program defining first the windows of each domains, with two domains. In that case, we consider only one parameter to make the illustration simplest. Figure 1 shows the result obtained for the domain windows.

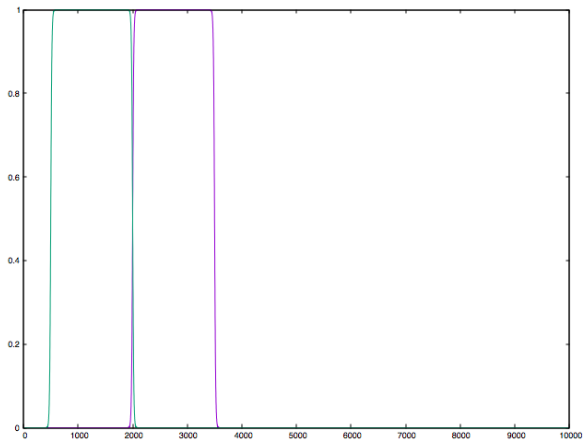

Figure 1

Once the domains defined, we can use them to create a parametrized impedance in two laws.

Let's take for example next impedance law:

$$Z\left(i, R, \alpha\right) = \mathcal{D}_1^i R i_1 + \mathcal{D}_2^i R\left(1 + \alpha i_1\right) i_1$$

Once defined this law, we can give i_1 all possible values in one context, for a given resistance R and for various values of the parameter α.

Previous equation was drawn and gives the figure 2. First domain creates only one curve as it doesn't depend on any parameter. At the contrary, second domain creates many curves that make the surface seen on the figure.

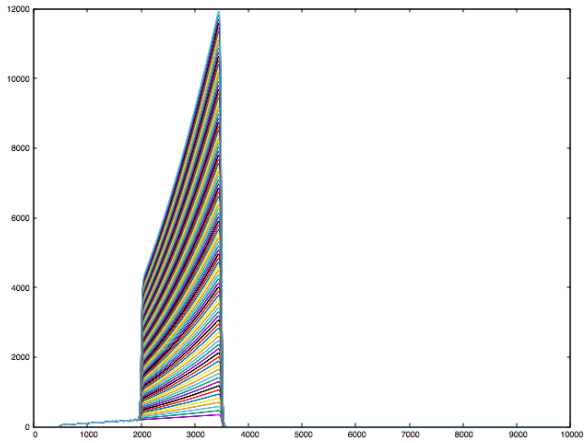

Figure 2

It's clear that depending on domain values, the manifold associated with the law can change radically of appearance.

4 Connecting primitive manifolds

Various of previous like manifolds can be glued in order to create more complex ones[5]. The only constraints is to make in accordance the domains. If we imagine two laws of definitions:

$$Z_1 = \mathcal{D}_1 Z_1^1 + \mathcal{D}_2 Z_1^2 \quad Z_2 = \mathcal{D'}_1 Z_2^1 + \mathcal{D'}_2 Z_2^2$$

If we want to make the direct summation of the two previous primitives, it gives:

$$Z = \mathcal{D}_1 Z_1^1 + \mathcal{D}_2 Z_1^2 + \mathcal{D'}_1 Z_2^1 + \mathcal{D'}_2 Z_2^2$$

It's clear that domains should be equal to point out same values of parameters coming from the environment conditions. Another possibility exists when domains complete each other. For example: $\mathcal{D}_1' + \mathcal{D}_{1[} = \mathcal{D}_1''$. Note that one of the domain limit must be open (private of the last value) in order to connect

it continuously with the other one. We may write: $\mathcal{D}_1'' = \mathcal{D}_1 \setminus [+\mathcal{D}_1'$. In both cases, direct summation becomes possible and has meaning. If one of the impedance law doesn't depends on limits, the direct summation results from the classical addition of the two original laws.

5 Impact on the second geometrization process

When impedance is written using such laws defined on domains (restricted ones, which should be always the case for real object), what is the consequence on the second geometrization process? In other words, how the metric will be expressed when these domaines come in square power?

We consider the system represented by the impedance matrix:

$$Z = \begin{bmatrix} \mathcal{D}_1 A + \mathcal{D}_2 B & C \\ C & D \end{bmatrix} \tag{5.1}$$

Note that even if a current threshold intervene in $\mathcal{D}$, it is simplest to choose another current than the current multiplied by this domain. In this case, when computing the derivation to extract base vectors, the result is directly the impedance operator itself. So, previous matrix leads to the jacobian:

$$J = \begin{bmatrix} \mathcal{D}_1 A + \mathcal{D}_2 B & C \\ C & D \end{bmatrix} \tag{5.2}$$

under the hypothesis that $A \dots D$ are all pure reals.

The metric coming from this jacobian is:

$$G = \begin{bmatrix} (\mathcal{D}_1 A + \mathcal{D}_2 B)^2 + C^2 & (\mathcal{D}_1 A + \mathcal{D}_2 B)\, C + CD \\ (\mathcal{D}_1 A + \mathcal{D}_2 B)\, C + CD & C^2 + D^2 \end{bmatrix} \tag{5.3}$$

Due to its decomposition in domains, the first term (without C^2) can be developed in:

$$(\mathcal{D}_1 A)^2 + (\mathcal{D}_2 B)^2 + 2\,(\mathcal{D}_1 A \cdot \mathcal{D}_2 B)$$

Now in case of perfect differentiation in the domain, i.e. $\mathcal{D}_1 \cdot \mathcal{D}_2 = \delta_{12} = 0$, the square is reduced to the summation of the squares of each impedance domain. But this case is rarely a physical one. More, it often leads to instability in numerical computations. So, if we use a common interval to both domains $\mathcal{D}_1$ and $\mathcal{D}_2$ as previously, the product $2\,(\mathcal{D}_1 \cdot \mathcal{D}_2)\, AB$ is different from zero. This term leads to a power P_δ linked with the transition phases when the material change of characteristics under environment parameters. If K and Q are the two mesh currents, we have for example:

$$P_\delta = 2\,(\mathcal{D}_1 \cdot \mathcal{D}_2)\, AB.K^2$$

So that the metric can be finally written: $G + G_\delta$, where G_δ integrates all the phase changes of the manifold. Note that these transition states belong to each primitive manifold and also to shared energy (extradiagonal element $G_{ij}, i \neq j$) if any impedance law is constructed under the domain formalism. Noting

$$\mathcal{D}_{ij} = \mathcal{D}_i \cdot \mathcal{D}_j \tag{5.4}$$

some interactions can disappear or not depending on the domain intersections, giving the metric a new meaning: it gives a deep description of the distance dependance versus the environment parameters. To solve quite easily the problems, it's clear that it's better when these parameters are not directly the flux themselves.

6 Curvature

In one domain, the law can integrate a function that's depends on one current. For example: $Z(\mathcal{D}) = \mathcal{D}_1 R + \mathcal{D}_2 R.K$. In that case, first base vector could be (in a three dimension space):

$$\mathbf{b}_1 = \begin{bmatrix} \mathcal{D}_1 R + \mathcal{D}_2 R.K \\ b_2 \\ b_3 \end{bmatrix} \tag{6.1}$$

where b_2 and b_3 are functions without currents.

Due to the component dependance with current, $\mathbf{b}_{11} = \partial_K \mathbf{b}_1$ exists. It means that Christoffel's coefficients $\Gamma_{11,i}$ can exist. What does it means ? It means that for some environment conditions, the base vector $\mathbf{b}_1$ change with current K value. It's component on

the local TpS tangent plan $\mathbf{b}_i$ are given by Christoffel's coefficient[7]. In other words, from one location to another $d\mathbf{b}_1 = \Gamma_{11,i}\mathbf{b}_i dK^i$.

If Christoffel's coefficients exist, this is a testimony that environment influence deeply the object behavior. The dependance of space with currents translates the fact that energy looks for a technique to compensate a kind of saturation in energy distribution. It translates also hysteresis phenomenons[6] in the system behavior. Remanent processes are involved and modify space characteristics that creates base vector evolution depending on the currents. In a general consideration, these effects always exist for any system. Linearities are approximations for limited domain definition of the impedances. It corresponds to limited definition of the manifolds.

7 Conclusion

Domains defined through logistic functions gives general and accurate definitions of systems, using impedance functions. It leads to manifold point of view of these systems where material disappear behind equations and their curves that gives a new view of the same systems. That was original Kron's idea[8]: to say that once equations are established from the circuit schematic, this last one wasn't useful at all. Working on equations and linked manifolds give all the available information to decide of actions of modification to apply to the circuit and to understand its behavior.

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