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DYNAMIC ASPECTS IN RECONSTRUCTABILITY ANALYSIS: THE ROLE OF MINIMUM UNCERTAINTY PRINCIPLES *

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Abstract

The role of principles of minimum uncertainty in dealing with the reconstruction problem of systems with dynamic properties is discussed. The aim of the reconstruction problem, one of two problems addressed by reconstructability analysis, is to determine the smallest possible sub-systems by which a given overall system can be adequately represented.

Résumé

Nous traitons du rôle des principes de moindre incertitude en analysant le problème de reconstruction de systèmes à caractère dynamique. Le but du problème de reconstruction, l'un des deux problèmes concernés par l'analyse de la reconstructibilité, est de déterminer les sous-systèmes minimaux capables de représenter de manière adéquate un système global donné.

This little paper is dedicated to Wyllis Bandler. In my opinion, Wyllis is a rather unusual mathematician in the sense that his research work has almost always focused on important but underdeveloped areas of mathematics. One area that has considerably been advanced by Wyllis' research is the area of mathmatical relations (Bandler and Kohout, 1980 *a*, *b*, 1986, 1987 *a*, *b*). It was primarily this area of research, which was of interest to both of us, that brought us together some 15 years ago. The aim of this paper is to illustrate

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the great generality and richness in the conceptualization of systems as mathematical relations. As such, it is a proper tribute to Wyllis' work.

The conceptual framework and terminology employed in this paper are fully presented in my recent book (Klir, 1985). I assume that the reader is familiar with this material on systems problem solving. In particular, I assume that he or she is familiar with fundamentals of reconstructability analysis; these are adequately covered in Chapter 4 of the mentioned book as well as in two overview articles (Klir and Way, 1985; Pittarelli, 1990).

In general, reconstructability analysis deals with problems associated with the relationship between systems perceived as wholes (overall systems) and their various subsystems (*i.e.*, parts of the whole). One of the problems addressed by reconstructability analysis, which is the subject of this paper, is a *reconstruction problem*. Given an overall system, the aim of the reconstruction problem is to determine the smallest possible subsystems by which the overall system can be adequately represented and, if desirable, reconstructed to an acceptable degree of approximation.

In general, the term *system* is used in reconstructability analysis for a set of variables together with some characterization of the constraint among these variables. Each variable of a system is viewed as an abstraction of some real-world attribute. It is associated with a finite set of *states* (values), each of which represents a class of appearances of the corresponding attribute.

Constraints among variables can be expressed in various ways. Reconstructability analysis has been developed for systems whose constraints are characterized in terms of mathematical relations defined on the Cartesian products of the state sets and in terms of probability and possibility measures defined on the relations. It is assumed that the constraint is *invariant* with respect to the backdrop against which the variables are observed, most often time, space, or a population of individuals of the same kind.

The reconstruction problem belongs to the general class of problems of systems simplification. The system is simplified by being broken down into appropriate subsystems. All simplification problems share the same basic principle: a sound simplification of a given system should minimize the loss of relevant information while achieving the required reduction of its complexity. This principle requires an appropriate measure of information for the mathematical framework within which the given overall system is conceptualized.

Well-justified measures of information are now available for the main mathematical formalisms currently utilized for conceptualizing systems (Klir and Folger, 1988). For example, when the system is conceptualized just in terms of a mathematical relation, the Hartley measure is known to be the

only applicable measure of information (Hartley, 1928; Klir and Folger, 1988; Rényi, 1970). When probability theory is employed to characterize the constraint among variables, the appropriate (unique) measure of information is the Shannon entropy (Klir and Folger, 1988; Shannon, 1948). Well-justified measures of information are now also available for systems conceptualized in terms of possibility theory (Dubois and Prade, 1986; Klir and Folger, 1988; Zadeh, 1978) and the Dempster-Shafer evidence theory (Klir and Folger, 1988; Shafer, 1976). In all these measures, the amount of information is measured in terms of the amount of reduced uncertainty. The more our total ignorance (full uncertainty) regarding states of relevant variables is reduced by the given system, the more information the system contains.

Simplification is not the only reason why it is desirable to represent overall systems by their subsystems. A discovery that a system can be represented by a specific set of subsystems may provide the investigator with some knowledge that is not available, at least explicitly, in the corresponding overall system. For example, the subsystem configurations may give him information about causal relationships, the significance of the individual variables, the strength of dependencies among them, etc. In general, this additional knowledge may help the investigator to develop a better insight into the nature of the phenomenon investigated.

A subsystem representation of a given overall system is particularly illuminating when dynamic properties are involved. This aspect of the reconstruction problem has been largely neglected in previous writings on reconstructability analysis (Klir and Way, 1985). My aim in this paper is to discuss some of the issues involved in the reconstruction problem of systems with dynamic properties. The issues are discussed in terms of a specific example.

Let me use an example of an ecological system that is adopted from my book (Klir, 1985; Ex. 9.5, pp. 445-449) to discuss some of the issues associated with the dynamic interactions among subsystems. The terminology employed in this discussion is also adopted from the book. The overall system in this example consists of six sampling variables defined in terms of three ecological variables by the mask specified in Figure 1. For the sake of simplicity, the

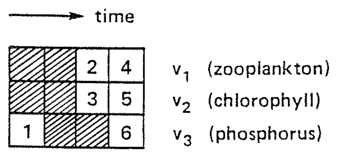

Figure 1. Mask of the overall system.

sampling variables are identified by the integers 1-6. The constraint among the variables, which is not given here, was expressed in terms of possibility theory; it was derived from data collected on Oneida Lake (in New York State) on a daily basis for 193 days in 1977. The system is neutral, which means that none of the three variables is an input variable. For our purpose there is no need to describe the overall system in more detail.

When the overall system was analyzed by reconstructability analysis, we obtained structure systems (*i.e.*, sets of subsystems) specified in Table 1.

Table 1. *Reconstruction hypotheses with minimum loss of information on refinement levels 1-10 (subsystems are identified by lists of sampling variables and are separated from each other by the slashes).*

Refinement level	Structure system
1	12346/12356
2	1246/1356/2346/2356
3	1356/2346/2356
4	1356/234/2356
5	1356/2356/34
6	1356/256/34
7	1356/25/34
8	156/25/34/356
9	16/25/34/356
10	16/25/34/36/56

These structure systems were determined as the best reconstruction hypotheses (*i.e.*, those minimizing the loss of information) on refinement levels 1-10. refinements beyond level 10 are not included because the loss of information was prohibitively large for them according to the criteria specified by the ecologist who studied the system.

We can see from the mask in Figure 1 that sampling variables 4, 5, 6 are generated (*i.e.*, determined by the constraint), while variables 1, 2, 3 are generating. The control of individual variables of the overall system can thus be expressed in terms of the block diagram in Figure 2, where the triangular blocks represent delays of a specified number of units of time (1 day is the unit of time in our example). The arrows on the interrupted lines indicate that the generated variables can be utilized, in principle, also as generating variables. That is, for example, variable 4 and 6 may contribute in determining variable 5. Allowing this possibility, however, may result in inconsistencies and, consequently, it is acceptable only in special cases.

Let us consider now some of the structure systems specified in the Table 1. First, let us consider the structure system at the first level of refinement

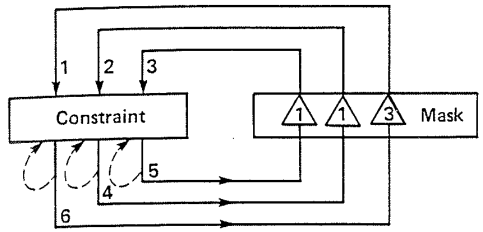

Figure 2. Block diagram of the overall system.

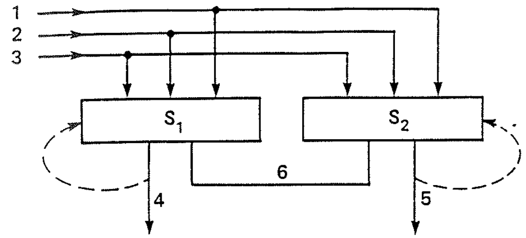

Figure 3. Structure system on the first refinement level.

12346/12356. Its block diagram is shown in Figure 3. For the sake of simplicity, the delays through which variables 1, 2, 3 are obtained are not shown in this figure.

Observe that each of the two subsystems S_1 and S_2 of the structure system depicted by the block diagram in Figure 3 contains at least one generated variable. If there were a subsystem with no generated variable in the structure system, such a subsystem would be totally useless and may be excluded. Observe further that the generated variable 6 is shared by both of the subsystems. Since only one of the subsystems can control the variable, we may conceptualize the structure system in one of two ways. The arbiter in deciding which of these two conceptualizations is preferable should be based on the generative (predictive) uncertainties associated with the generated variables. Normally, we should try to minimize these uncertainties. Let us discuss the two options in detail.

As mentioned previously, the given overall system is formulated in this example in terms of possibility theory. That means that a possibilistic measure of uncertainty (and information) must be used. This measure is known in the literature as the U-uncertainty (Higashi and Klir, 1983; Klir and Folger, 1988). Clearly, it must be applied in appropriate conditional forms. Let $U(X|Y)$ denote the conditional U-uncertainty associated with state set X of the generated variable provided that states of the state set Y of the generating variables are employed as conditions. It is known that

$$U(X|Y)=U(X, Y)-U(Y),$$

Where $U(X, Y)$ and $U(Y)$ denote the uncertainty associated with the state set of all variables and with the state set of generating variables, respectively.

For convenience, let symbols 6_1 and 6_2 denote that the control of variable 6 is assigned to subsystems S_1 and S_2, respectively. Furthermore, let the form of each conditional U-uncertainty be identified by the variables involved. For example, let $U(6_2|1235)$ denote the U-uncertainty of variable 6 when generated by the subsystem S_2.

If the given overall system were expressed in terms of a mathematical formalism different from possibility theory, we would obviously have to replace the U-uncertainty with another measure of uncertainty, pertinent to the formalism employed. However, everything else in our discussion would remain the same.

The following are the conditional uncertainties involved in the two control alternatives of variable 6:

(6_1) variable 6 is controlled by subsystem $S_1 - U(4|123)$, $U(5|1236_1)$, $U(6_1|1234)$;

(6_2) variable 6 is controlled by subsystem $S_2 - U(4|1236_2)$, $U(5|123)$, $U(6_2|1235)$.

In general, we should compare total uncertainties involved in these two alternatives. This amounts to comparing the sums of the uncertainties representing each of the alternatives. Since, however, generated variables need not be equally important, it may often be desirable to assign appropriate weights to the individual uncertainties. Hence, we may define functions $p(6_1)$ and $p(6_2)$ for the two alternatives by the formulas

$$p(6_1)=w_4 U(4|123)+w_5 U(5|1236_1)+w_6 U(6_1|1234),$$
$$p(6_2)=w_4 U(4|1236_2)+w_5 U(5|123)+w_6 U(6_2|1235),$$

where w_4, w_5 and w_6 are the weights (e.g., numbers in the closed interval [0,1]) expressing the importance of variables 4, 5 and 6, respectively. Values $p(6_1)$ and $p(6_2)$ are nonnegative and the larger they are, the less desirable the respective alternatives are. It is thus reasonable to view the values as penalties associated with the two alternatives. Clearly, we select the alternative with the smaller penalty.

Let us analyze now, in a similar fashion, the structure system on refinement level 8 (Table 1). Its block diagram is shown in Figure 4. We observe the

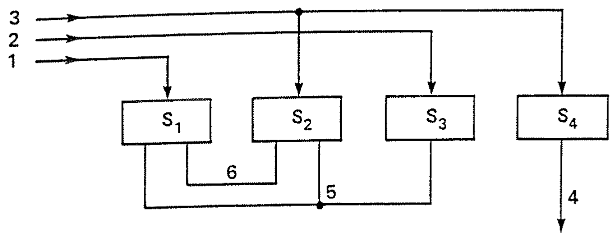

Figure 4. Structure system on the eighth refinement level.

following facts:

– each of the four subsystems $S_1 - S_4$ contains at least one of the generated variables and, consequently, none of them can be immediately recognized as superfluous, even though some may become superfluous under specific assignments of the controls of the dependent variables;

– variable 4 is contained only in subsystem S_4 and hence, its control is unique;

– variable 5 is shared by three subssystems (S_1, S_2, S_3), and this offers three alternatives of its control;

– variable 6 is shared by two subsystems (S_1 and S_2) and can thus be controlled by either of them.

It follows from these facts that there are six control alternatives in this case. The following are their characterizations and the associated penalties (the same notation is used as in the previous example):

(5_1 & 6_1) Subsystems S_2 and S_3 are superfluous and we have

$$p(5_1 \& 6_1)=\min[p_1(5_1 \& 6_1), p_2(5_1 \& 6_1)],$$

where

$$p_1(5_1 \& 6_1)=w_4 U(4|3)+w_5 U(5_1|16_1)+w_6 U(6_1|1),$$
$$p_2(5_1 \& 6_1)=w_4 U(4|3)+w_5 U(5_1|1)+w_6 U(6_1|15_1);$$

(5_1 & 6_2) subsystem S_3 is superfluous and

$$p(5_1 \& 6_2)=\min[p_1(5_1 \& 6_2), p_2(5_1 \& 6_2)],$$

where

$$p_1(5_1 \;\&\; 6_2) = w_5\,\mathrm{U}(5_1\,|\,16_2) + w_6\,\mathrm{U}(6_2\,|\,3),$$
$$p_2(5_1 \;\&\; 6_2) = w_5\,\mathrm{U}(5_1\,|\,1) + w_6\,\mathrm{U}(6_2\,|\,35_1);$$

$(5_2 \;\&\; 6_1)$ subsystem S_3 is superfluous and

$$p(5_2 \;\&\; 6_1) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_2\,|\,36_1) + w_6\,\mathrm{U}(6_1\,|\,15_2);$$

$(5_2 \;\&\; 6_2)$ subsystems S_1 and S_3 are superfluous and

$$p(5_2 \;\&\; 6_2) = \min\,[p_1(5_2 \;\&\; 6_2),\; p_2(5_2 \;\&\; 6_2)],$$

where

$$p_1(5_2 \;\&\; 6_2) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_2\,|\,36_2) + w_6\,\mathrm{U}(6_2\,|\,3),$$
$$p_2(5_2 \;\&\; 6_2) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_2\,|\,3) + w_6\,\mathrm{U}(6_2\,|\,35_2);$$

$(5_3 \;\&\; 6_1)$ subsystem S_2 is superfluous and

$$p(5_3 \;\&\; 6_1) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_3\,|\,2) + w_6\,\mathrm{U}(6_1\,|\,15_3);$$

$(5_3 \;\&\; 6_2)$ subsystem S_1 is superfluous and

$$p(5_3 \;\&\; 6_2) = w_1\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_3\,|\,2) + w_6\,\mathrm{U}(6_2\,|\,35_3).$$

Again, we select the alternative with the smallest penalty.

Observe that in the control alternatives $(5_1 \;\&\; 6_1)$ and $(5_2 \;\&\; 6_2)$ of the previous example, only one of the generated variables, 5 or 6, is allowed to participate in the determination of the other one. Although these variables may simultaneously influence each other in special cases of relationships (with no inconsistencies), such cases seem extremely rare and I do not consider them in this paper for the sake of simplicity.

Let me analyze one additional structure specified in Table 1, the last one (at refinement level 10). Its block diagram is given in Figure 5. We can easily see that this structure system offers again six alternatives of control of the generated variables (this time two alternatives for variable 5 and three alternatives for variable 6). Block diagrams for these six alternatives are shown, without the superfluous subsystems, in Figure 6. The following are the penalties associated with the alternatives:

$$p(6_1 \;\&\; 5_3) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_3\,|\,6_1) + w_6\,\mathrm{U}(6_1\,|\,1);$$
$$p(6_1 \;\&\; 5_4) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_4\,|\,2) + w_6\,\mathrm{U}(6_1\,|\,1);$$
$$p(6_2 \;\&\; 5_3) = w_4\,\mathrm{U}(4\,|\,3) + w_5\,\mathrm{U}(5_3\,|\,6_2) + w_6\,\mathrm{U}(6_2\,|\,3);$$

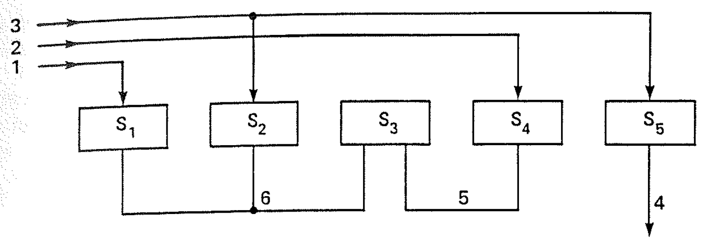

Figure 5. Structure system on the tenth refinement level.

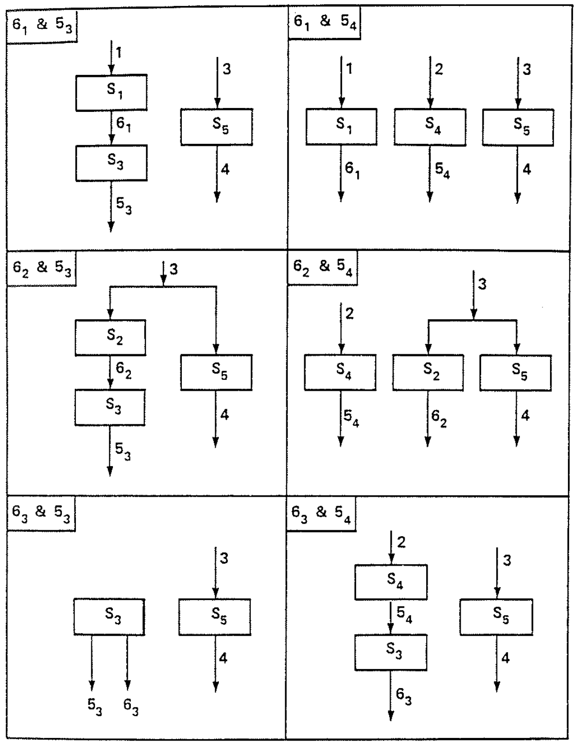

Figure 6. Six alternatives of control of generated variables for the structure system specified in Figure 5.

$$p(6_2 \ \& \ 5_4) = w_4 \, \mathrm{U}(4 \mid 3) + w_5 \, \mathrm{U}(5_4 \mid 2) + w_6 \, \mathrm{U}(6_2 \mid 3);$$

$$p(6_3 \ \& \ 5_3) = \min \, [p_1(6_3 \ \& \ 5_3), \, p_2(6_3 \ \& \ 5_3)], \qquad \text{where}$$

$$p_1(6_3 \ \& \ 5_3) = w_4 \, \mathrm{U}(4 \mid 3) + w_5 \, \mathrm{U}(5_3 \mid 6_3) + w_6 \, \mathrm{U}(6_3),$$

$$p_2(6_3 \ \& \ 5_3) = w_4 \, \mathrm{U}(4 \mid 3) + w_5 \, \mathrm{U}(5_3) + w_6 \, \mathrm{U}(6_3 \mid 5_3);$$

$$p(6_3 \ \& \ 5_4) = w_4 \, \mathrm{U}(4 \mid 3) + w_5 \, \mathrm{U}(5_4 \mid 2) + w_6 \, \mathrm{U}(6_3 \mid 5_4).$$

As in the previous case, we again select the alternative with the smallest penalty.

I could continue now to analyze the remaining structure systems specified in Table. I believe, however, that the three discussed examples are sufficient to illustrate the key issues involved. Methods for dealing with these issues have not been sufficiently developed as yet. This paper should be viewed as a stimulus for further research of this topic rather than as a finished product.

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