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A mathematical analysis
of the map of mental states

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et définissant les facteurs de personnalité indépendamment du cas particulier de l'homme.

Le fonctionnement d'un ordinateur est volontairement démotivé et désaffectivé, justement pour lui permettre de se plier aux motivations et aux demandes affectives de l'utilisateur. Lorsque Kasparov interrompt une partie d'échecs pour en continuer l'étude sur l'ordinateur, il impose à la machine ses propres motivations, sa propre affectivité. Comme j'ai déjà tenté de le montrer, la comparaison entre intelligence artificielle et humaine est faussée au départ du fait que la dimension historique est habituellement négligée. Une même réflexion peut être faite concernant la motivation ou la vigilance.

On peut aisément concevoir des machines, certes peu utiles mais démonstratives, dont le fonctionnement est associé à des motivations. Il suffit d'accroître, comme dans les tortues de Grey Walter, le rôle d'interfaces avec l'environnement et d'orienter ensuite le fonctionnement opératoire central principalement vers les données d'interface. Une motivation apparaîtra immédiatement, comme pour les tortues de Grey Walter dont toute l'activité se concentrait sur un retour au chargeur de batteries lorsque ces batteries commençaient à faiblir. Un minimum de réflexion sur l'action apparaît alors.

On peut également envisager un ordinateur qui ne mette pas en jeu en permanence la totalité de ses besoins, par exemple pour faire une économie d'énergie ou éviter un échauffement très important. Des critères faciles à définir, peuvent apprécier l'importance et l'urgence des problèmes et régler la puissance mise en jeu. A chaque instant, l'ordinateur présenterait un état «émotif» caractérisé par la richesse des mécanismes en action. On peut concevoir alors un état de forte émotion, marqué par un conflit entre le choix d'une réponse rapide à haut coût énergétique et une réponse lente économique, ou encore entre une réponse rapide seulement probable et une exploration tactique complète très exigeante en temps et en puissance. Les ordinateurs connaîtront peut-être demain des «jours de colère».

A MATHEMATICAL ANALYSIS OF THE MAP OF MENTAL STATES

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Abstract

The domains of variability of main mental variables for the Clark's map have been established. The equations of spirals lines, being the trajectories of normal mental life, have been introduced. An attempt has been made to describe the map as a dynamical system. Two proximity measures of mental states have been defined, which correspond to two ways of transition from a mental state to another one. Finally, a mathematical study of these distances has been done and some conclusions about the transition velocity between two states have been proposed.

Résumé

Les domaines correspondant aux principales variables mentales de la carte de Clark ont été établis. Nous avons explicité les équations des lignes spirales, qui sont les trajectoires de la vie mentale ordinaire. Ensuite, on a essayé de présenter la carte comme un système dynamique. Deux mesures de proximité entre les états mentaux ont été définies. Elles correspondent à deux cas possibles de transition d'un état mental à un autre. Enfin, l'étude mathématique de ces distances a été faite et quelques conclusions sur la vitesse de transition entre les états mentaux ont été proposées.

1. Introduction

In his book (John Clark, 1985) has described a map of mental states, a map of the mind. Various maps of the mind have been considered earlier (the Buddhist Wheel of Life, for example), but this one seems to be the most systematic. The advantage of maps, in general, is that they sum up a lot of

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information in a reduced form. By contrast, their disadvantage is that many things in the region being mapped cannot be included in a particular map. In the mentioned book, the mind has been defined as the whole range of mental states, i.e. of conscious states, sleeping and waking, which a person can experience. Clark's map displays them on the surface of a cone, or equivalently, of a disc.

By "mental state" Clark understands the values taken by a set of main mental variables at any particular time. We shall consider the following main variables : mind work, general mood (pleasant or unpleasant), intensity of general mood, attention and things, concentration and diffusion of attention.

Informally, these variables are defined as follows:

- mind work: represents our level of mental alertness, or mental energy,
- we usually say that our general mood is either pleasant or unpleasant. It also varies in intensity. At ordinary "average" levels of intensity, unless we are in an unpleasant general mood, we don't notice our mood whatsoever,
- attention and things: attention itself is a sort of mental currency. We "pay" attention to things. Sometimes we pay more attention to one thing and less to another.
- concentration and diffusion of attention: when our attention is high (low) in comparison with the number of things to which it is being paid, it is called concentrated (diffuse).

The variety and subtlety of mental life requires adding some extra variables, such as anxiety, obsessions, phobias and so on, and some special variables as attachment, enlightenment etc. (see Clark, 1985), which, however, won't be considered in this work. Clark's map allows us to plot mental states in ordinary life as well as the mystical path, mental illnesses and effects of drugs (Clark, 1985). Ordinary life is interpreted very widely, in the belief that it contains a wide enough variety of mental states, some of which can be very strange. There are of course certain border, intermediate zones between ordinary life and other three types of mental life : mystical states, mental illnesses and drug states, as commented in (Clark, 1985).

This work will deal with mental states in the ordinary life only. Its main purpose consists in introducing some calculations to the original geometrical map in order to be able to do some numerical simulations. More exactly: while Clark's model is descriptive, giving only main ideas of mental variables and mental states and geometrical representation of them, in this work I try to introduce some formalization of above notions. I define domains of variability, introduce the equations of the spiral lines, define the map as a dynamical

system, introduce two proximity measures of mental states: the rectilinear and the curvilinear ones.

The paper will be organized as follow. First, we shall define main mental variables by determining their domains of variability. Then, the equations of Clark's spiral lines will be introduced, taking into account his two rules (dependences between some of variables). Next, an attempt will be made to describe the map as a dynamical system by introducing a set of states and a transition function. A differential equation such that the spiral lines are its particular solutions will be given, too. A study of mental states proximity measures will be performed; namely, for two admissible ways of transition between mental states, two measures of distances will be defined. Some conclusions about the transition velocity between two mental states will be drawn. Finally, remarks about the perspectives of such a numerical analysis of the map will be shown.

2. Mathematical Description of the Map

As it has been mentioned in the Introduction, the map of mental states can be represented, according to (Clark, 1985), as a disc, being a projection of a cone (see *fig .1*). The main variables, listed above, are introduced as follows:

- r (radius of the disc) represents a mind work (then, a big disc while waking, and a small one while sleeping),
- θ (angle) is a measure of intensity of general mood ($\theta \in [0, \pi]$) represents a pleasant mood and $\theta \in [\pi, 2\pi]$ - an unpleasant one),
- ρ (distance of the representative point Ψ from the origin O), gives the number of things in the mind at a particular time; $\rho \in [0, r]$.
- $x = r - \rho$ represents, according to Clark's Rule 1 (see Clark, 1985), a measure of attention units,
- $c = x/\rho = r/\rho - 1$ is a concentration ratio (Clark's Rule 2, Clark, 1985); $c > 1$ means that the representative point Ψ is in a concentrated zone (more attention than things), and $c < 1$ means that Ψ is in a diffuse zone (less attention than things),
- z is the zero-state, i.e. intensity = zero, concentration ratio = 1.

Considering the mind work r as a parameter, the mental state (representative point Ψ) is completely defined by the couple (ρ, θ) . Let M be a set of such mental states. It is possible to draw some lines on the map, as shown in figure 2; more exactly, lines I, II, III and IV (see Clark, 1985). These lines are projections of spiral lines on the surface of the cone onto its base. Hence, for simplicity we shall call them spiral lines, too.

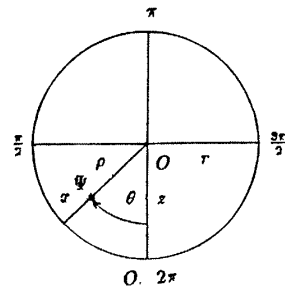


Figure 1. The map and its main variables.

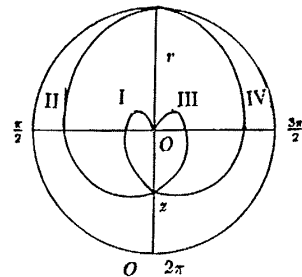


Figure 2. The spiral lines on the map.

These spiral lines represent an equation which makes the intensity level proportional to the difference between things and attention units. Such a definition of the spiral lines is well motivated (Clark, 1985); particularly, such a spiral line represents the simplest relationship between things and attention.

Mathematically it means, that

$$\theta = \frac{\pi|\rho - x|}{r} = \frac{\pi|2\rho - r|}{r}. \quad (1)$$

For a pleasant mood, i.e. when $\theta \in [0, \pi]$, there are two possibilities for the values ρ can take:

- a) when $\rho \leq r/2$ (i.e. $\rho \leq x$), there is less things than attention units and it means that we are in the concentrated zone. Then, we have

$$\theta = \frac{\pi(r - 2\rho)}{r}, \quad (2)$$

(i.e. spiral line I). One can see easily that, in particular cases: if $\rho = r/2$, then $\theta = 0$ (zero-state z); if $\rho = r/4$, then $\theta = \pi/2$; if $\rho = 0$, then $\theta = \pi$, as shown in figure 2.

- b) when $\rho \geq r/2$ (i.e. $\rho \geq x$), there are more things than attention units and we are in the diffuse zone. Then, equation (1) becomes:

$$\theta = \frac{\pi(2\rho - r)}{r}, \quad (3)$$

(i.e. spiral line II). One can see, that: if $\rho = r/2$, then $\theta = 0$ (zero-state z); if $\rho = 3r/4$, then $\theta = \pi/2$; if $\rho = r$, then $\theta = \pi$, as shown in figure 2.

For an unpleasant mood, i.e. when $\theta \in [\pi, 2\pi]$, there are two possibilities as well:

- c) when $\rho \leq r/2$ (concentrated zone), then

$$\theta = \frac{\pi(2\rho + r)}{r}, \quad (4)$$

(i.e. spiral line III). It is obvious, that: if $\rho = r/2$, then $\theta = 2\pi$ (zero-state z); if $\rho = r/4$, then $\theta = 3\pi/2$; if $\rho = 0$, then $\theta = \pi$, as shown in figure 2.

- d) when $\rho \geq r/2$ (diffuse zone), then

$$\theta = \frac{\pi(-2\rho + 3r)}{r}, \quad (5)$$

(i.e. spiral line IV). Obviously: if $\rho = r/2$, then $\theta = 2\pi$ (zero-state z again); if $\rho = 3r/4$, then $\theta = 3\pi/2$; if $\rho = \pi$, as shown in figure 2.

The axis symmetry between spiral lines I and III, II and IV, respectively, which can be seen in figure 3 means that, if to some value ρ_1 corresponds some value θ_1 on the spiral line I (II), then, to the same value ρ_1 corresponds the value $2\pi - \theta_1$ on the spiral line III (IV). Because of this symmetry, the numerical analysis will be made for a pleasant mood only, i. e. for the spiral lines I and II.

3. The Map as a Dynamical System

In this section, we shall try to see the map as a dynamical system, i.e. as a pair (M, Φ) , where M is a set of states (mental states) and Φ is a transition function which determines the transfer from state to state. Such an approach can be useful for further studies, because it allows us to apply all known methods of system analysis; for example, the problem of stability. It is rather clear how to choose the set of states M . For each level of mental work (a fixed value of the parameter r) the mental state (system state) Ψ ,

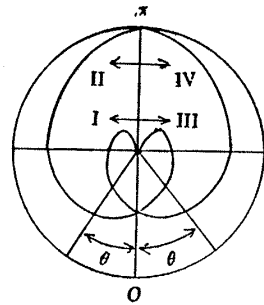


Figure 3. The axis symmetry between spiral lines.

is completely defined by ρ and θ . Thus, M is the set of pairs (ρ, θ) , where $0 \leq \rho \leq r$ and $0 \leq \theta \leq 2\pi$, defining all the points lying in the disc of radius r . The problem, however, needs some further explanation. First, we suppose all "normal" mental states, i.e. the states appearing in the ordinary life, lying on one of the four spiral lines defined below (states lying on spiral lines I and III are concentrated zone states, and those on II and IV are diffuse zone states). Next, we suppose that in a neighbourhood of each spiral line there exist some distinguished zones (zones, A, B, C and D, as shown in figure 4). Mental states belonging to them are due to mental illnesses or drugs, according to Clark's suggestions. For example, C is a distaste zone, i.e. zone of delusion and hallucinations and D is a zone, where the degree of concentration is higher than "necessary" for the intensity value given. In the same way, suitable interpretation can be given for the zones A and B. We suppose that all other zones are the No-Go areas, i.e. the representative point (mental state) cannot go there. And, finally, we suppose a "discontinuity" between the permitted zones except for the possible transition through, the zero-state z . Thus, the representative point Ψ has to "jump" the No-Go zones. This problems will be studied mathematically in the next section.

Now, let us define the transition function Φ for the states lying on the spiral lines I and II (for III and IV by symmetry), i.e. for the mental states in ordinary life, as mentioned in the Introduction. If ρ varies from ρ_1 to $\rho_1 + \Delta\rho$, then θ goes from θ_1 , to $\theta_1 + \Delta\theta$, according to equation (1)

$$\theta_1 + \Delta\theta = \frac{\pi |2\rho_1 + 2\Delta\rho - r|}{r}, \quad (6)$$

which implies that

$$\Delta\theta = \frac{\pi (|2\rho_1 + 2\Delta\rho - r| - |2\rho_1 - r|)}{r} \quad (7)$$

on each of the spiral lines I or II. Particularly, on the spiral line I, where $2\rho_1 + 2\Delta\rho \leq r$ and $2\rho_1 \leq r$, we obtain

$$\Delta\theta = -\frac{2\pi \Delta\rho}{r}. \quad (8)$$

On the spiral line II, $2\rho_1 + 2\Delta\rho \geq r$ and $2\rho_1 \geq r$, so

$$\Delta\theta = \frac{2\pi \Delta\rho}{r}. \quad (9)$$

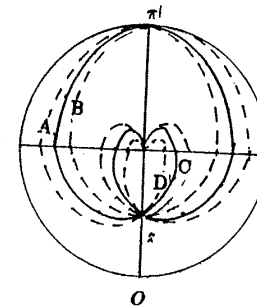


Figure 4. Permitted zones on the map.

In the case of allowable transitions between two states lying on two different spiral lines (for example, the initial state Ψ_1 , on I, and the final one Ψ_2 on II), we have to calculate $\Delta\theta_1$ (from θ_1 to z) first, and then, $\Delta\theta_2$ (from z to Ψ_2), which corresponds to the transition through the zero-state z . Finally, $\Delta\theta = \Delta\theta_2 - \Delta\theta_1$, as shown in figure 5.

Now, we can introduce the transition function Φ as follows. Let I be the outside world; for our purposes it is considered as a set of time functions, expressing real processes in the outside world, i.e.

$$I = \{u_{tt'} | u_{tt'} : [t, t'] \rightarrow \mathbb{R}^n, t' \geq t\}. \quad (10)$$

We consider here every time-function $u_{tt'}$ as a map of an interval $[t, t']$ into \mathbb{R}^n (multidimensional process). Usually (Bertalanffy, 1973, Mesarovič

and Takahara, 1975), Φ is defined as a map :

$$\Phi : I \times M \rightarrow M, \quad (11)$$

which determines the state s' at time t' as a function of the state s at time t and the input (outside world process) $u_{tt'}$ over the interval $[t, t']$. Let us suppose now that the outside world modifies ρ (things), and θ (intensity) as a function of ρ (see the equation (6)), s' is a final result of this modification. According to the above results, s' is given by

$$s' = (\rho', \theta') = \Phi(u_{tt'}, s) = \Phi(u_{tt'}, \rho, \theta) \quad (12)$$

with

$$\rho' = \rho + \Delta\rho, \quad (13)$$

where $\Delta\rho$ is, in fact, an unknown function of $u_{tt'}$. The nature of the dependence of $\Delta\rho$ on $u_{tt'}$ remains the most difficult, unsolved question. However, answering this question is not the purpose of this work which consists in studying some properties of the map. The second component of (12), according to (6), is given by

$$\theta' = \theta + \frac{\pi(|2\rho + 2\Delta\rho - r| - |2\rho - r|)}{r} \quad (14)$$

with two particular cases corresponding to (8) and (9).

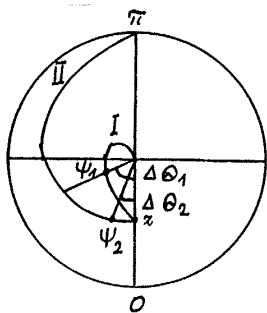


Figure 5. Transition through the zero-state z .

In order to define the map as a dynamical system, another approach is possible as well. As we cannot say anything about the outside world influence on the transition between the mental states, we can define the map (a dynamical system) as a pair (M, Γ) , where M , as above, is the set of mental states and Γ is the family of trajectories of trajectories on M . In our simplest case (the ordinary live only), Γ contains four trajectories, i.e. the spiral lines I, II, III and IV. There are two states (zero-state z and (r, π) -state), lying on more than one trajectory. In other words, starting from one of these states, there are some bifurcations possible, i.e. some different spiral lines to take. In order to preserve causality principle, basic for dynamical systems (Mesarovič, Takahara, 1975), we must suppose that there exists a real parameter which governs these bifurcations. Unfortunately it seems, at present, impossible to define this parameter and, then, to apply the bifurcation theory to find the exact conditions of these bifurcations (Hassard, 1981; Marsden, 1976). Instead, it can be useful to find a differential equation such that the four spiral lines are particular solutions. If we consider ρ (things) as an independent variable, and θ as a function of ρ , we can verify that this equation has the following form:

$$\frac{d\theta}{d\rho} = \frac{2\theta}{|2\rho - r|}. \quad (15)$$

Obviously, as the dependence between the outside world and things, i.e. between $u_{tt'}$ and ρ remains unknown, we are not able to write down explicitly the differential equation with t (time) as an independent variable and the intensity θ as its function.

4. Measures of Proximity of Mental States

As has been mentioned above, there are two possibilities of a transition between the spiral lines I and II. The first one consists in the "jumps" between the spirals, as mentioned in the previous section, and as is shown in figure 6. Such a jump represents a rapid transition from state $\Psi_1 = (\rho_1, \theta_1)$ to $\Psi_2 = (\rho_2, \theta_2)$.

It is too early to understand in what exactly consists the difference between a jump and a transition through z . It seems, however, that the transitions through the zero-state z correspond to mental-state changes in ordinary life and under the influence of some average signals coming from the outside world. On the other hand, jumps take place between two, opposite enough, states as results of some unusual input signals (for example, between a concentrated zone state and a diffuse zone one, or between a pleasant mood state and an unpleasant one, and so on). That's why it is necessary to define two different

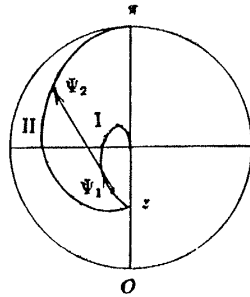


Figure 6. Jumps between the spiral lines.

proximity measures, corresponding to these two possible ways of transition between mental states.

Now, we assume for both cases that the duration of a jump (or a transition) is directly proportional to the distance between the states. Thus, as a measure of this duration we can use the corresponding distance. Let us first calculate the rectilinear, euclidean distance between two states: $\Psi_1 = (\rho_1, \theta_1)$ and $\Psi_2 = (\rho_2, \theta_2)$, Ψ_1 lying on the spiral line I and Ψ_2 on the spiral line II. This distance d_r , corresponding to the jump between Ψ_1 and Ψ_2 — d_r , can be calculated as :

$$d_r = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos(\theta_2 - \theta_1)}, \quad (16)$$

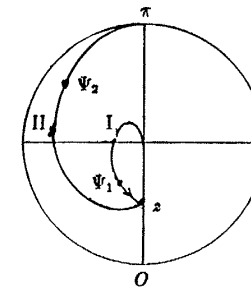
where $\theta_1 = \pi(r - 2\rho_1)/r$, $\theta_2 = \pi(2\rho_2 - r)/r$ (see equations (2) and (3)). Hence, finally

$$d_r = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos \frac{2\pi(r - \rho_1 - \rho_2)}{r}} \quad (17)$$

is the rectilinear proximity measure of two mental states Ψ_1 and Ψ_2 . One can see easily that $d_r \geq |\rho_1 - \rho_2|$ and, in the particular case, $d_r \geq |\rho_1 - \rho_2|$ iff $\rho_1 + \rho_2 = r$, i.e. iff O , Ψ_1 and Ψ_2 are colinear.

The second possible transition between the same pair of states: Ψ_1 and Ψ_2 is through zero-state z , as shown in figure 7.

In this case, the proximity measure is a curvilinear distance l which is equal to $l_1 + l_2$; l_1 is the length of the part of the spiral line I calculated from Ψ_1 to z , and l_2 is the length of the part of the spiral line II calculated from z to Ψ_2 . Both lengths can be found using classical integral calculus methods. We obtain

Figure 7. Transition through zero-state z .

$$l_1 = \frac{r((\theta_1 - \pi) \sqrt{(\theta_1 - \pi)^2 + 1} + \pi \sqrt{\pi^2 + 1} + \operatorname{arsinh}(\theta_1 - \pi) + \operatorname{arsinh} \pi)}{4\pi}, \quad (18)$$

$$l_2 = \frac{r((\theta_2 + \pi) \sqrt{(\theta_2 + \pi)^2 + 1} - \pi \sqrt{\pi^2 + 1} + \operatorname{arsinh}(\theta_2 + \pi) - \operatorname{arsinh} \pi)}{4\pi}. \quad (19)$$

Thus, the curvilinear proximity measure is given by

$$l = \frac{r((\theta_1 - \pi) \sqrt{(\theta_1 - \pi)^2 + 1} + (\theta_2 + \pi) \sqrt{(\theta_2 + \pi)^2 + 1} + \operatorname{arsinh}(\theta_1 - \pi) + \operatorname{arsinh}(\theta_2 + \pi))}{4\pi}. \quad (20)$$

We can see that greater is the mind work r , longer is the length l and, thus, the transition time t between both corresponding mental states. We are able to distinguish two particular cases:

– a) $\theta_1 = \theta_2 = O$; in this case, of course, $l_1 = l_2 = l = O$

– b) $\theta_1 = \theta_2 = \Pi$; in this case l_1 becomes the length s_1 of the whole spiral line I (i. e. from z to (O, π)), and l_2 becomes the length s_2 of the spiral line II (from z to (r, π)). We find, respectively:

$$s_1 = \frac{r(\pi \sqrt{\pi^2 + 1} + \operatorname{arsinh} \pi)}{4\pi} \quad (21)$$

and

$$s_2 = \frac{r(2\pi\sqrt{4\pi^2+1} - \pi\sqrt{\pi^2+1} + \operatorname{arsinh} 2\pi - \operatorname{arsinh} \pi)}{4\pi}. \quad (22)$$

It is clear that the lengths of both spiral lines depend on the mind work r only.

Let us now change the variables in the following way: $\theta_1 - \pi = t_1$, $\theta_2 + \pi = t_2$ (thus, $t_1 \in [-\pi, 0]$ and $t_2 \in [\pi, 2\pi]$). Using new variables t_1 and t_2 , we express l_1 and l_2 as:

$$l_1 = \frac{r(t_1\sqrt{t_1^2+1} + \pi\sqrt{\pi^2+1} + \operatorname{arsinh} t_1 + \operatorname{arsinh} \pi)}{4\pi}, \quad (23)$$

$$l_2 = \frac{(t_2\sqrt{t_2^2+1} - \pi\sqrt{\pi^2+1} + \operatorname{arsinh} t_2 - \operatorname{arsinh} \pi)}{4\pi}. \quad (24)$$

The derivatives of l_1 with respect to t_1 and of l_2 with respect to t_2 define the velocities of distance changes from the zero-state z to the $(0, \pi)$ or (r, π) -states, respectively

$$\frac{dl_1}{dt_1} = \frac{r\sqrt{t_1^2+1}}{2\pi}, \quad (25)$$

$$\frac{dl_2}{dt_2} = \frac{r\sqrt{t_2^2+1}}{2\pi}. \quad (26)$$

Both of them are increasing functions and then they assume maxima at the ends of respective intervals: dl_1/dt_1 at $t_1 = -\pi$ and dl_2/dt_2 at $t_2 = 2\pi$. Returning to the variables θ_1 and θ_2 , we obtain the points of the maxima of velocities:

$$-\pi = \theta_{1max} - \pi \leftrightarrow \theta_{1max} = 0, \quad (27)$$

$$2\pi = \theta_{2max} + \pi \leftrightarrow \theta_{2max} = \pi. \quad (28)$$

The following conclusion can be drawn from the above results: along the spiral line I the most rapid change of the distance from the zero-state z , considered as a function of θ , takes place in the neighbourhood of z , while along the spiral line II the same occurs near the (r, π) -state.

5. Final Remarks

Among the weak points of the Clark's map, mentioned by his author (Clark, 1985), the most important is the fact that in many situations, in mental illnesses for example, it is necessary to add some extra variables. The reason for this fact is the simplicity of the map which has very few main variables. Thus, for a more advanced study, the map has to be enriched, i.e. some new variables (dimensions) have to be added. But such an operation will complicate the map and make it harder to use. Nevertheless, a construction of a multidimensional map seems to be one of the nearer purpose of our work. Moreover, more exact mathematical analysis should be done, focused on many well-known mental states, such as: The Void, Mystical State Proper, Peak Experience, Enlightened States and so on (see Clark, 1985). Their exact positions on the map should be established, and the corresponding distances between them calculated. Finally, our hypothesis that the duration of both types of transition is proportional to the distance between the states should be experimentally verified in order to justify the model.

Acknowledgement

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