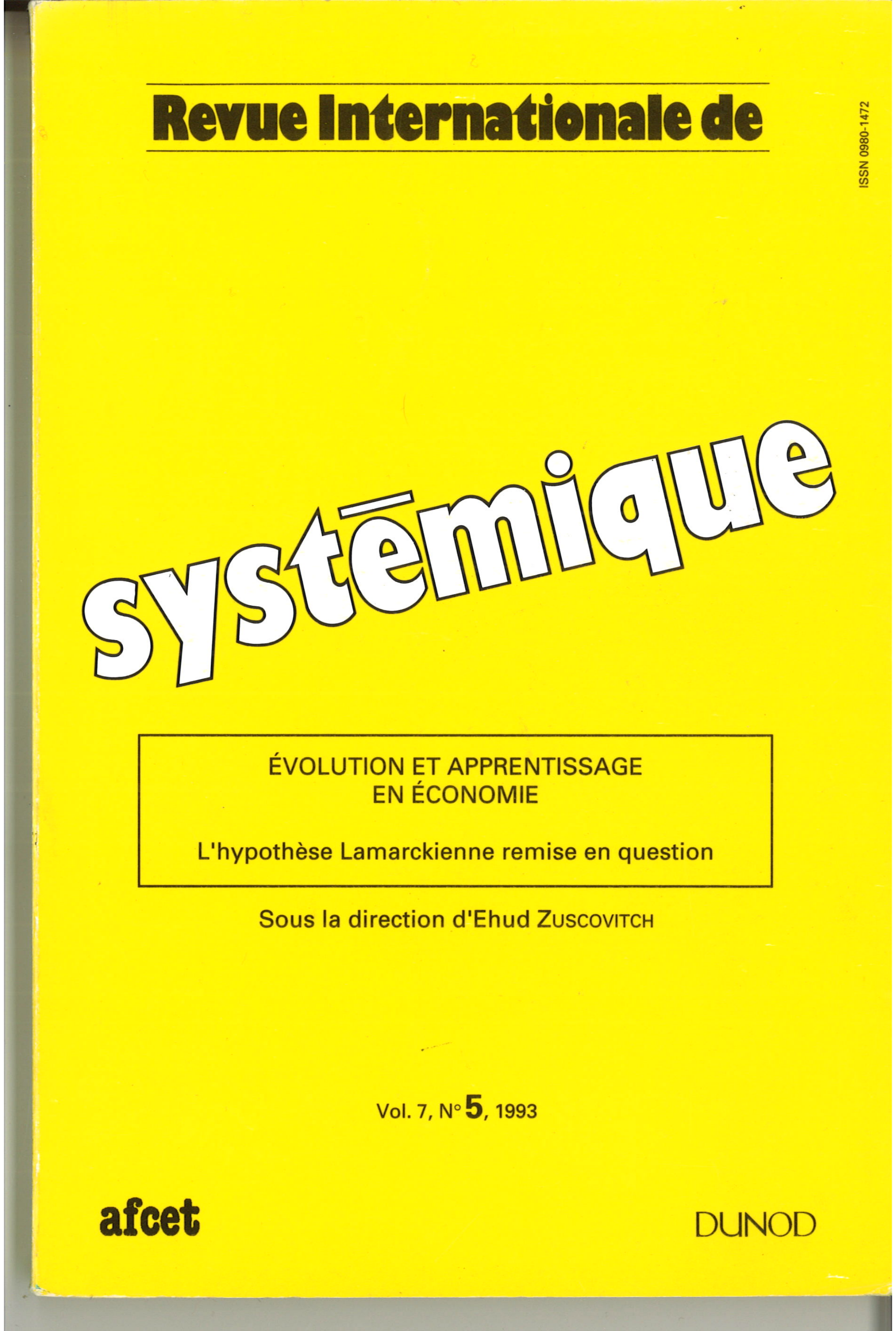

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Evolution and forward induction in game theory

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EVOLUTION AND FORWARD INDUCTION IN GAME THEORY

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Abstract

Forward induction criteria require perfectly rational players and common knowledge of the structure of the game. Selection-mutation processes, in contrast, work with players who undergo random shocks, have a limited rationality and a limited information on the structure of the game, which is repeated many times. Yet both lead to a similar equilibrium selection. This paper goes into this result, and thereby tackles the interface between rationality and evolution.

Résumé

L'emploi de critères d'induction projective en théorie des jeux nécessite des acteurs parfaitement rationnels, pour qui la structure du jeu est connaissance commune. Les critères de type sélection-mutation, au contraire, reposent sur des hypothèses de rationalité ou d'information limitées ; les acteurs, myopes et soumis à des chocs aléatoires, n'ont qu'une connaissance partielle du jeu, supposé répété un grand nombre de fois. Malgré ces divergences, les deux types de critères conduisent à une sélection similaire d'équilibres. L'article analyse ce résultat et aborde ainsi les connexions entre rationalité et évolution.

I. INTRODUCTION

Rationality can arise from the evolution of a society, whose individuals have a limited rationality. This is not a new idea, it is perhaps even commonplace. Yet it is still troubling, especially when it leads to a connexion between two new topics in game theory, both developed over the last decade, which have, at first sight, no common points: *forward induction* and *selection-mutation processes*.

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So, on the one hand, forward induction relies on the *common knowledge* of the structure of the game and on (*perfectly*) *rational* players, who are able to infer all the information conveyed by a sequence of past actions. On the other hand, selection-mutation processes tackle games where the players have a *limited rationality,* undergo *random shocks* that lead them to an erratic behavior, and have only a *partial information* on the structure of the game.

Yet, oddly enough, these strongly different topics lead, in many games, to a *convergent equilibrium selection.* This striking fact calls for a more detailed examination. Indeed, if convergence proves to occur not only in very specific examples, then the study backs the claim that rationality-based criteria can be used in not perfectly rational contexts.

Hence we are led to tackle the interface between, evolution and rationality. We do not intend to be exhaustive: the paper is just an introduction, aimed to encourage further study. So, we begin, in section 2, by briefly presenting the most salient features of forward induction and selection-mutation processes. Next, in section 3, we expand some examples which stress the nature of the convergence. The latter is discussed in detail in section 4. We conclude, in section 5, on the notion of learning, which is behind the behavior evolution of not perfectly rational individuals.

II. FORWARD INDUCTION AND SELECTION-MUTATION PROCESSES

The purpose of this section is not to survey the numerous papers on forward induction (see Kohlberg, 1989, Umbhauer, 1991 *b*, Van Damme, 1990) and evolution processes (see, for a partial study, Maynard Smith, 1982, Hofbauer & Sigmund, 1988). We only stress the features of both topics, which help to work out the convergence.

In short, *forward induction* requires that each player, when playing, takes into account all the information conveyed by the past observed actions, which are, as far as possible, supposed to be rational.

The main feature of forward induction is that it moves away from the notion of *momentary insanity* (see Selten, 1975), which underlies the *backward induction* principle, that is behind the still most employed equilibrium concepts (*e.g.* Perfect Equilibrium (Selten, 1975), Sequential Equilibrium (Kreps & Wilson, 1982)). According to the backward induction principle a past deviation (*i.e.* a past out of equilibrium action) is always viewed as a temporary error, which never affects the future behavior of the player. In other terms, errors are supposed to be not correlated in time, and, as a result, deviations are never strategic. In contrast, forward induction, by expecting each action to be rational, allows deviations to be strategic. Now a deviation can convey meaningful information; it may *signal* something on the nature of the deviator and his future behavior, causing the other players *to act accordingly.* It follows that a deviation can be fruitful for the deviator, the result being that some (backward induction) equilibria do not stand up to forward induction.

Yet forward induction is not a *precise* concept. This justifies the existence of a huge number of criteria which try to capture it. Very roughly, they fall into three classes. The first one (rather mathematical) is an attempt to construct, *ex nihilo,* new equilibrium concepts, on the basis of a certain number of requirements, among others, forward induction; the Stable Set concept (Kohlberg & Mertens, 1986), is the well knownst (and forerunner) criterion of this class. The two other classes adopt a more interpretative way of doing. The idea is to refine existing equilibrium comcepts (Perfect Equilibrium, Sequential Equilibrium...) with the aid of forward induction. Yet the two classes differ in their way of interpreting the out of equilibrium actions. One class (see Banks & Sobel, 1987; Cho, 1987; Cho & Kreps, 1987; Grossman & Perry, 1986; Kohlberg, 1989) looks for the meaning of out of equilibrium actions, without calling in question the meaning of the equilibrium ones (this features is also implicit in Kohlberg & Mertens, 1986). The other one (see Mailath, Okuno-Fujiwara & Postlewaite, 1993; 1991 *a*), in contrast, puts forth the idea that, if playing an out of equilibrium action conveys a signal, then not playing it may also convey a signal, which can be different from the planned one. It follows that a signal, to be meaningful, has to be *consistent* with *all* its implications (*even on equilibrium actions*), causing equilibria to only be dismissed by other equilibria. This last class will be the closest to the selection-mutation processes.

We do not aim, in this short paper, to retrace the history of *evolution processes,* all the more we are mainly interested in the latest in date, *i.e.* the *selection-mutation processes.* What matters is that the contexts, which gave rise to the evolution processes, are biological populations, whose individuals are not rational at all. Yet the study of the evolution of these populations points out the survivance of the fittest (and only them). So, even if no individual is rational, even if nobody solves a conscious optimization problem, all happens as if the individuals were rational. Of course, in (*very over-simplified*) biological societies, the solution of the puzzle is that fitter individuals have a higher reproduction rate. Hence, if the offspring is identical to the parents, the fittest individuals will, over the long run, displace the less fit ones (for more sophisticated evolutions, see Hofbauer & Sigmund, 1988).

Game contexts are of course not biological ones. Yet the biological models bore fruits: the most important one is the idea that a strategy, which leads to a high payoff today, should be played more often tomorrow. This idea, which stems from the higher reproductive rate of fitter individuals, underlies the *selection processes*. It makes only sense if the game is played a lot of (infinite number of) times, and if players have a limited rationality. Indeed, it supposes short-sightedness, as players switch *tomorrow* to *today* best replies, without taking into account that tomorrow's state of play, due to the switches, will differ from today's one.

The well-knownst dynamical equations that are in accordance with a selection process are the replicator equations. According to these equations, the probability of playing a pure strategy grows (decreases) continuously in time, if the payoff it ensures is higher (lower) than the mean one. Other dynamical systems have been analysed (see Friedman, 1992 for a survey).

We recall the systems proposed by Samuelson (1991) and Young (1993), as we will go back to them later on.

Samuelson (1991) is interested in a smooth adaptation system, in which each player, from period t to period $t+1$, does not move with probability close to one, and switches to the today best reply with the complementary probability.

Young (1993) (see also Canning, 1992) studies a selection process, where the players' memory exceeds one period. Hence a player switches to the best reply to a history of states (of play). What is more, Young (1993) supposes that each player switches to the best reply to a sample of k states, drawn from the set of the last m states; (k/m) denotes the player's information incompletness.

All selection processes share a common point: the state to which they lead over the long run *strongly depends on the initial state* of play. Following common practice we call *absorbing sets* the smallest sets of states of which the process can not break out, once it is at one of these states. We call *basin of attraction* of an absorbing set, the set of states such as the selection process, when being at these states, leads with probability one to the absorbing set (see Samuelson, 1991 for more details). Thus, in other terms, the (over the long run) observed absorbing set strongly depends on the initial state of play. It follows that there is no room for an innovative behavior.

In order to allow more innovation in the players' actions, *mutation processes* are added to selection processes. Introduced in the late 1980's (see Foster & Young, 1990; Kandori, Mailath & Rob, 1993; Samuelson, 1991; Young, 1993), a mutation process leads each player, when playing, to act, with a small but strictly positive probability, in accordance with a random process. The way in which the mutation process is added to the selection process (after or simultaneously) does not matter (see *e.g.* Samuelson, 1991 and Young, 1993 for two different ways). What matters is that the support of the random process is the whole player's strategies' set, causing every action to be played with a strictly positive probability.

Stated technically, thanks to the mutation process, there are only strictly positive probabilities in the transition matrix (from one period's states of play to the following period's ones). As a result, there exists a unique stationary probability distribution over the strategies, which *does not depend on the initial state of play*. More precisely, Samuelson (1991) and Young (1993) establish that the absorbing sets that will be observed (with a probability significantly different from 0) over the long run, have the following property: the sum of mutations needed to reach their basin of attraction, from every other absorbing set, is minimal.

To throw light on this result (due mainly to Freidlin & Wentzell, 1984) we give the intuition behind it (largely out of Foster & Young, 1990's paper). An essential feature of the mutation process is that mutations, which act like perturbations, can *cumulate*. More precisely, the study of the stability of rest points in deterministic dynamical systems introduces random shocks whose effects do not interfere. In contrast, the mutation process generates a *sequence* of perturbations, *the impact of one perturbation being not necessarily ended when a new perturbation appears*. Hence the effects of the perturbations can cumulate, which allows *to cross the frontiers* between the basins of attraction (all frontiers can be crossed because of the full support of the mutation process). This explains why there exists a unique stationary probability distribution and why it does not depend on the initial state of play. Yet some crossings are *more difficult* than others, in that they require *more mutations*. As the probability of a sequence of mutations is an exponential decreasing function of the number of mutations involved, the easily reachable absorbing sets will be observed more often over the long run than the others. Samuelson, 1991 and Young, 1993's result follows.

III. EXAMPLES

We expand 3 examples, which will illustrate most of the comments given in section 4.

For a start, look at the game depicted in figure 1.

There are two Sequential Equilibrium *paths*[1] E_1 and E_2 characterized by:

E_1: player 1 plays m_1 regardless of type and player 2 plays r_1 after m_1.

E_2: player 1 plays m_2 regardless of type and player 2 plays r_2 after m_2.

Only E_2 is a Consistent Forward Induction Equilibrium Path (CFIEP hereafter, see Umbhauer (1991 *a*)), that is to say a forward induction criterion based on the notion of consistent signal. Its associated set of outcomes (to simplify we call it also E_2) is the only Stable Set (*i.e.* Kohlberg & Mertens's, 1986 mathematical forward induction criterion). At last, E_2 is the only absorbing set, which is observed over the long run (with probability close to one), when applying a selection-mutation process (Samuelson, 1991 and Young, 1993).

We refer to the authors for greater details on the different criteria. Here we only stress the points which will allow us to yield some insights into the reasons of the convergence in equilibrium selection.

So E_1's outcome is not Stable[2] because, if t_2, due to the perturbations, deviates more to m_2 than t_1, then player 2 is *a priori* incited to play r_2 after m_2. To prevent this (E_1 upsetting) response, t_1 should deviate more than is expected by the perturbations. But, to do this, he should be indifferent between m_1 (which ensures him a payoff equal to 2) and m_2, which is clearly impossible.

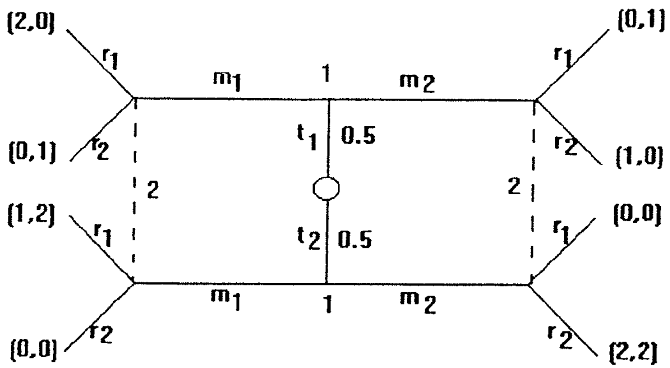

Figure 1. It is a signaling game. Player 1 can be of two types, t_1 and t_2; his type is drawn randomly according to an a priori *probability distribution (0.5 for each type), which is common knowledge of player 1 and player 2. Player 1 knows his type when playing: he chooses a message, m_1 or m_2. Player 2 observes the message, but not the type (hence the information sets (dashed lines)), and then chooses a response, r_1 or r_2. The game ends with this action. The payoffs are given in the column vectors (the first (second) coordinate is player 1 (2)'s payoff).*

E_1 is not a CFIEP because of the system of beliefs that assigns m_2 to both types with the *a priori* probabilities. Indeed it leads player 2 to play r_2 after m_2. As a result, t_2 is incited to play m_2, causing player 2 to assign m_1 to t_1 and hence to play r_2 after m_1 (he still plays r_2 after m_2). It follows that t_1 plays m_2 too, which validates the system of beliefs. Hence E_1 is upset (it is displaced by E_2).

E_1 is upset by the selection-mutation processes, in that one mutation is enough for reaching E_2's basin of attraction, when starting in E_1's absorbing set, the converse being false. Indeed look at the *(limit)* E_1 Sequential Equilibrium, in which player 2 plays r_2 after m_2 with probability 1/2. Introduce one mutation, one more agent of player 2[3] playing r_2 after m_2. It sets off a selection process, *which is similar to the process involved by the above CFIEP system of beliefs,* and which leads to E_2's absorbing set. This process is: t_2 switches to m_2, player 2 plays r_2 after m_2 and m_1, and t_1 finally plays m_2.

Now look at he game depicted in figure 2.

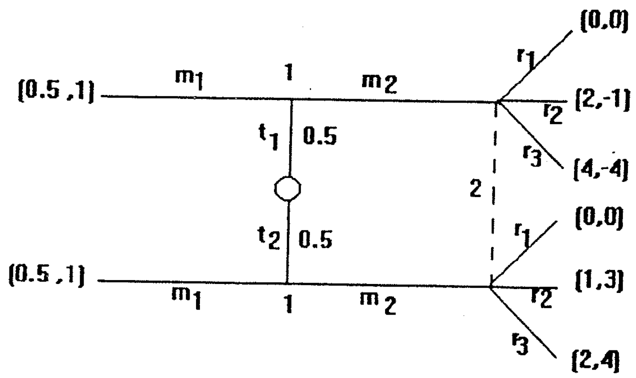

Figure 2. Similar to figure 1's legend.

Consider the Sequential Equilibrium paths, E_1 and E_2, characterized by:

E_1: player 1 plays m_1 regardless of type.

E_2: player 1 plays m_2 regardless of type, and player 2 plays r_2 after m_2.

E_1 ('s outcome) is Stable; in contrast, E_1 is not a CFIEP and it does not stand up to a selection-mutation process.

E_1 is Stable in that small perturbations can not upset it. Indeed, if the perturbations are such as t_1 deviates more to m_2 than t_2, then player 2 plays r_1 after m_2, which supports E_1. In the other case, player 2 is incited to play r_3 after m_2. To prevent this (E_1 upsetting) response, t_1 has to deviate more than is required by the perturbations; to do this, he has to be indifferent between m_1 and m_2, which is possible.

E_1 is not a CFIEP because of the system of beliefs that assigns m_2 to both types with the *a priori* probabilities. This system leads player 2 to play r_2 after m_2, causing both types of player 1 to play m_2, which validates the system. Hence E_1 is upset (it is displaced by E_2).

E_1 is also upset by, for example, Young's (1993) selection-mutation process. Suppose, when starting in E_1, that one agent of type t_2 "mutates" to m_2. This mutation implies the following selection process: player 2 plays r_3 after m_2, which leads both t_1 and t_2 to play m_2, causing finally player 2 to play r_2. So one mutation is enough for reaching E_2's basin of attraction when starting in E_1's absorbing set. The converse is false. This game has one more absorbing set, which is also at one mutation from E_2's basin of attraction, the converse being false. This completes the proof by ensuring that E_2 is observed with probability close to 1 over the long run.

Finally look at the game depicted in figure 3:

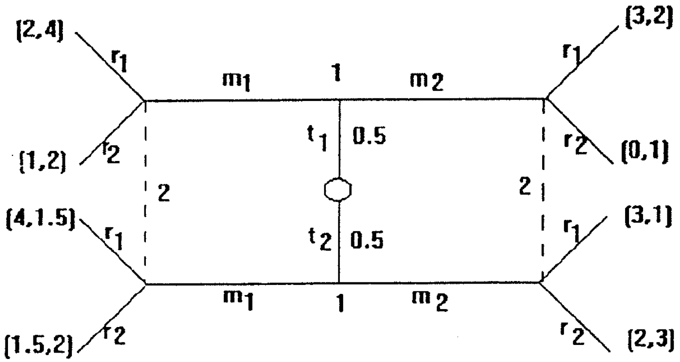

Figure 3. Similar to figure 1's legend.

This game has 2 Sequential Equilibrium paths, E_1 and E_2, characterized by:

E_1: player 1 plays m_1 regardless of type and player 2 plays r_1 after m_1.

E_2: t_1 (t_2) plays m_1 with probability 1/7 (4/7), m_2 with the complementary probability, player 2 plays r_1 with probability 5/13 (6/13) after m_1 (m_2), r_2 with the complementary probability.

E_1 ('s outcome) is not Stable. Yet E_1 is a CFIEP and it is not upset by the selection-mutation processes.

E_1 is not Stable because, if t_1, due to the perturbations, deviates more to m_2 than t_2, then player 2 is led to play r_1 after m_2. To prevent this (E_1 upsetting) action, t_2 has to play m_2 more than required by the perturbations. Yet, to do this, he should be indifferent between m_1 (which ensures him a payoff equal to 4) and m_2 (which ensures him at most a payoff equal to 3), which is clearly impossible.

In contrast E_1 is a CFIEP, because there does not exist a consistent system of beliefs which upsets E_1.

Neither is E_1 upset by a selection-mutation process. Indeed, one mutation is not enough for reaching E_2's basin of attraction when starting in E_1's absorbing set, whereas the converse way only requires one mutation. To capture why one mutation is not enough for leaving E_1's absorbing set, consider the E_1 *limit* Sequential Equilibrium where player 2 plays r_1 with probability 2/3 after m_2. Introduce one mutation of player 2, such as one more agent pays r_1 after m_2. It follows a selection process, that indeed, intuitively, in a first time, begins by leading the system away from E_1; however, in a second time, it leads the system back to E_1. Indeed the different steps of the selection process are: t_1 plays m_2, player 2 plays r_1 after m_2 and r_2 after m_1, t_2 plays m_2, player 2 plays r_2 after m_2, t_1 plays m_1, player 2 plays r_1 after m_1, and t_2 plays m_1. Hence the introduced mutation is not sufficient to break out of E_1's absorbing set (any other mutation would not allow to leave E_1's absorbing set either).

IV. DISCUSSION

The above examples do of course not prove the convergence in equilibrium selection. Yet they are sufficiently general to prove that the observed convergence is not a chance event. Thus there is a link between forward induction criteria and selection-mutation processes; in this section we yield some insights into this link, and, thereby, into the relationship between evolution and rationality.

It turns out that both the selection process and the mutation process are critical to the convergence. But, before discussing their joint impact, it is worth recalling that a selection process exhibits more rationality than is commonly believed (this rationality partly explains the convergence between selection processes and backward induction criteria (see Bomze, 1986; Van Damme, 1987).

Indeed, as already discussed in (for example) Kandori, Mailath & Rob (1993) and Samuelson (1991), a myopic behavior is not irrational if the system evolves slowly (replicator equations and Samuelson, 1991), as, as a result, tomorrow does not stronly differ from today. A slow evolution may by itself be rational, in that it may protect the system from overshooting phenomena, which hasten a system from a bad (*i.e.* a low outcome) state to another bad state, without ever leading it to a good state. Limited memory and incomplete sampling (Young, 1993) act similarly. Incomplete sampling allows players to not reply to the same play's states, which can prevent the system from being stuck in suboptimal cycles. Limited memory can allow to forget more fastly bad states, and hence allow to reach more fastly a good state.

We now go into the joint selection-mutation process.

To understand the critical role of mutations (selection processes alone would not lead to the same equilibrium selection) the best is to consider them as experimentations, rather than perturbations. Hence, when a player "mutates", he tries out a new behavior. Recall now that the forward induction principle leads the players to look for new (*i.e.* out of equilibrium) *strategic* actions, which can make them better off; if such actions exist they will be played. Hence, according to the forward induction principle, players also try out new behaviors, but only those which can increase their payoff. The mutation process by itself does not fit this condition of rational investigation, as it is a random process. Yet *infinite repetition* and *full support* ensure that the players will *necessarily* try out *all* the actions, including those which make them better off. Hence infinite repetition and full support remedy for the absence of rationality.

Yet, it is not enough to reach an interesting path, one has to stay on it. That is the role of the selection process. In many ways, the selection process works like a *consistency process*. Indeed, it consists of the sequence of best replies involved by a state of play. Hence a mutation allows to leave an equilibrium path (more generally an absorbing set) only if the sequence of best replies, implied by this mutation, leads to a new equilibrium path (more generally to a new absorbing set). Otherwise, one concludes that *one* mutation is not enough for the system to leave the tested path. Forward induction criteria based on the notion of consistent signal proceed on much the same way (this can readily be observed by looking at the three above games). In contrast, the other forward induction criteria do not share this way of doing, which explains the different equilibrium selection in the last studied game.

Let us make an additional comment on mutations. Mutations are not *"conservative"*, which can explain the difference between the Stable Set selection and the selection-mutation one, in the game depicted in figure 2. Indeed, the Stable Set criterion aims to *sustain* the tested set of outcomes. Simply put, for any set of small perturbations, one seeks to keep the tested set, by looking for perturbed actions which are in support of it. The mutation process does not share this way of doing: there is no wish to come back to the (due to the mutations) left absorbing set. The CFIEP shares this "unconstraint" investigation. Indeed, it also studies the possible meaning of out of equilibrium actions, without trying to sustain the tested equilibrium path.

To summarize, the preceding comments especially underline the convergence between forward induction criteria based on the notion of consistent signal and selection-mutation processes. They show why a random process with full support, coupled with a myopic best reply process, can lead to the same equilibrium selection than forward induction criteria (based on perfectly rational players and common knowledge) providing that the game is repeated an infinite number of times. Perhaps paradoxically, this result justifies a more systematic use of forward induction criteria in economy, a field where agents are not perfectly rational, nor perfectly informed of the structure of the situation in which they evolve. Indeed, as forward induction criteria lead to the same results as criteria based on random search and limited rationality (which seem better adapted to economic situations), there is no call to not use the first ones (all the more it is sometimes more easy to work with them).

Yet the nice convergence of (especially) the CFIEP criterion and the (Samuelson, 1991 and Young, 1993) selection-mutation processes, in the examples of section 3, can not lead to a outburst of enthousiasm, at least for the three following reasons. To begin with, the CFIEP criterion, contrary to the selection-mutation processes, can not eliminate perfectly mixed equilibrium paths, for lack of out of equilibrium actions. Next, the CFIEP criterion and the selection-mutation processes are not defined for the same games: the CFIEP definition is restricted to signaling games, whereas the (Samuelson, 1991 and Young, 1993) selection-mutation processes apply to more general games, but with finite strategies' sets. Finally forward induction criteria and selection-mutation processes contrast strongly in the time needed to reach the selected equilibria: the first criteria compute them instantaneously, whereas the second ones may require a very long time for the equilibria to arise. Hence the convergence only exists when the time horizon is long enough to allow the game to be repeated many times.

V. CONCLUSION

This paper is just an introduction. We only threw light on some convergence properties between forward induction and evolution processes. If this led us to speak about rationality, we never went into the *concept of rationality*. We merely employed the standard (game theoretical) definition, by which a player is rational if he takes into account all his information to maximize his payoff.

Nor did we discuss the related concept of *learning*.

At first sight, learning only makes sense in the selection-mutation processes (forward induction implies instantaneous learning). We follow Young (1993) in pointing out that learning needs not occur at the player's level. Indeed selection-mutation processes never suppose that the agents playing over time are the same. More precisely, the agents playing at a period may disappear (die) after playing; hence they can not improve their behavior over time, they can not learn. Yet, this will not prevent the society from improving the observed state over time. Hence learning occurs at the society level, but not necessarily at the individual one.

Yet we are quite aware that the notions of rationality and learning much richer concepts. So, for example, learning can not be reduced to the adaptation to (or searching for) a best way of behavior in a *given pattern of behaviors*. Learning and rationality should allow to improve on the behavior process itself. To give an example, they should not only lead the players to test new actions, but they should also drive them to test new selection processes, which may not necessarily be based on myopic best replies.

Going into these considerations would lead us far beyond the scope of the paper. So, to conclude, we only recall some works in game theory (or related topics) which tackle some of the issues raised above. Swinkels (1990) underlines a problematic feature of the Evolutionary Stable Strategies concept (see Maynard Smith, 1982); according to this concept, a rule of behavior is dismissed if it is not able to resist to a small number of agents switching to a new behavior. Yet, the new rule of behavior may itself be dismissed in the same way. Is it then rational for the players to change their first way of playing? In cooperative game theory, the notion of Bargaining Set (see, for example, Aumann & Maschler, 1964) tackles a similar problem. At last, it may be fruitful to pay attention to the literature on rational conjectural variations, which deals with the learning of the *way* in which the agents react (see Ulph, 1982 for a survey).

Notes and references

1. An equilibrium path consists of the actions which are observed if the equilibrium is played.

2. Roughly, a Set of outcomes is Stable if, for each perturbed game (in normal form) of the original game (in normal form), one gets a Nash equilibrium whose outcome is close to an outcome of the tested set.

3. In evolution processes, a player is considered as a set of agents, each agent playing a pure strategy.

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EVOLUTION STRATEGIES SIMPLE "MODELS" OF NATURAL PROCESSES?

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Abstract

Optimisation algorithms imitating nature have proved their usefulness in various domains of applications. Annealing processes, central nervous systems and biological evolution in turn have lead to independent optimisation methods. The field of evolutionary computation comprises genetic algorithms, genetic programming, evolutionary programming and *evolution strategies* on which this paper will focus. Because these algorithms can also serve as simple models of the underlying natural processes, why not forget about the problem solving capabilities for a moment and put the emphasis on self-adapting behaviour? And why not translate these results to other domains, thus stressing the similarity of the notions 'self-organisation' and 'evolution' and their usefulness a common descriptive language across the scientific disciplines?

Résumé

Depuis quelques années, les algorithmes d'optimisation (globale) imitant certains principes de la nature, ont pu démontrer leur utilité dans des domaines d'application variés. *Annealing processes*, le système nerveux central et l'évolution biologique ont chacun conduit à des méthodes d'optimisation indépendantes. Le domaine de calcul évolutionniste comprend les algorithmes génétiques, la programmation génétique, la programmation évolutionniste et les stratégies évolutionnistes sur lesquelles cet article sera focalisé. Puisque ces algorithmes peuvent aussi servir de modèles simples pour les processus naturels qu'ils représentent, pourquoi n'oublions-nous pas leur capacité à résoudre des problèmes afin d'étudier leur comportement auto-adaptatif ? Et pourquoi ne pas étendre ces résultats à d'autres domaines, soulignant ainsi la similitude de notions telles que « *self-organisation* » and « *evolution* » ainsi que leur utilité en tant que langage descriptif commun à plusieurs disciplines scientifiques ?

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