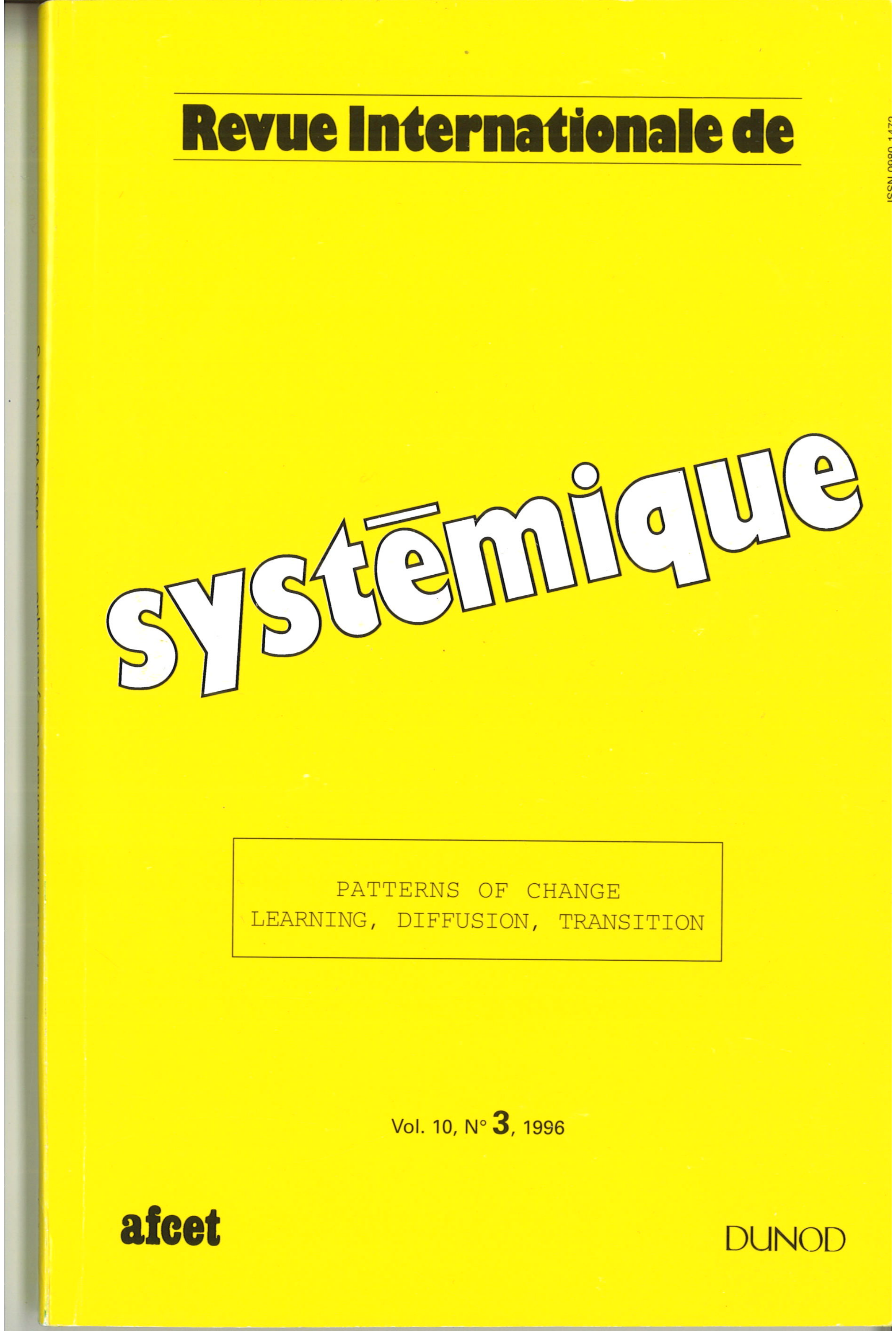

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MICRODIVERSITY AND MACRO-ORDER: TOWARD A SELF-ORGANIZATION APPROACH*

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Abstract
This paper reports on an emerging paradigm in evolutionary economics: self-organization. Though self- organization models do not constitute a unified theory it is still possible to read some basic standards by generalizing what might be considered as the methodological and theoretical key notions. Self-organization processes in the field of growth analysis is illustrated through a simple heuristic model. The main idea is to link a deterministic modelization to a probabilistic one in a master equation approach. This allows us to interpret deterministic equations as an average macro evolution based on stochastic micro-diversity.

Résumé
Cet article est consacré à un paradigme émergent au sein du courant évolutionniste : l'auto-organisation. L'auto-organisation ne constitue pas une théorie unifiée, il est toutefois possible d'en dégager dans un premier temps quelques grandes caractéristiques méthodologiques et analytiques. Dans un second temps, on considère une application de l'auto-organisation à l'analyse de la croissance fluctuante à partir d'un modèle illustratif simple. L'idée centrale consiste à relier modélisation déterministe et stochastique à partir d'une approche en terme de master équation. Dans ce schéma, les relations déterministes renvoient alors à une évolution des grandeurs macro-économiques moyennes, fondée sur une micro-diversité de nature stochastique.

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I. INTRODUCTION

A fundamental question raised by modern evolutionary economics concerns the understanding of the role played by microscopic diversity in both the emergence and evolution of macroscopic regularities. Among the different candidates to frame an understanding of the emergence and evolution of macroscopic structures from micro diversity the so-called "self-organization paradigm" appears to be a very promising device. This paper attempts to characterize self-organization (Section 2), and then illustrates some new topics of research concerning the relations between growth and self-organization (Section 3) [1].

This issue seems to acquire more and more analytical content as empirical –inductive– studies demonstrate the importance of procedural rationalities of economic agents in a complex environment. Interacting forms of learning, routine and innovative behaviours may in particular underlie an explanation of the remarkable adaptive potential and viability of capitalism. Such an explanation would be based upon theoretical non linear combinations, e.g. feedback effects, of *a priori* simple evolutionary mechanisms such as creation of micro-diversity and adaptation by selection and/or diffusion at a macroscopic level [*cf.* e.g. Dosi, Winter, 1994; Metcalfe, 1994; Witt, 1992].

To the extent that such "universal" mechanisms exist and may preside over many taxonomies or specifications, one must look for some axiomatic foundations on which to build theories of economic systems in evolution. The so-called appreciative methodology (Nelson, Winter, 1982; Nelson, 1987) of evolutionary analysis leads to such a program since, in a first step, it completes the necessary empirical generalisations or stylised facts concerning macroeconomic regularities and microeconomic behaviours (*cf.* e.g. Kaldor, 1986; Coricelli *et al.*, 1991). This recent material shall, in principle, provide enough reasonable assumptions and plausible causalities to model –on a deductive mode– how market economies came into being, are maintained and evolve. To proceed this way should clearly avoid both building "rococo logical palaces on loose empirical sand" (Nelson, Winter, 1982, p. 33) and succumbing to scientific monism.

II. SELF-ORGANIZATION AS AN APPRECIATIVE PARADIGM

Self-organization refers to the study of the global behaviour of complex dynamic systems as it results from the interaction of a large number of sub-

units. As such this paradigm encompasses many phenomena ranging from natural sciences to social ones (*cf.* e.g. publications of Cerisy conferences 1988 or Day, Chen, 1993). Though it does not constitute a unified theory it provides some methodological guidelines and formal tools which have been applied to economic systems. Before going on to characterize some of those, it is worthwhile to note that economic self-organization is more than a superficial "biological analogy". Its foundations are to be found at the intersection of non-linear dynamics and the theory of systems (*cf.* e.g. Lesourne, 1991; Silverberg, 1988). Thus, biological self-organization is no more a physical analogy than economics is a biological one; for all phenomena have their own characteristics but some do share common features, namely to be systems composed of interacting sub-units. Still, it is possible to distinguish different categories of self-organization processes depending on the nature of the intentionality associated to the systems.

Combining for instance previous typologies proposed by Atlan (1991) and Coricelli, Dosi, Orsenigo (1991), we can first distinguish a "weak form" of self-organization from a "strong form". The "weak form" corresponds to an exogenous intentionality devoted to the system. It is for instance the case of self- organizing neural networks in artificial intelligence for visual form. If, in this case, there exists a non-computed emergence of specific structures in the network due to generic computed learning rules, the purpose of the system –to recognise written letters e.g.– is still assigned from outside. On the contrary, the strong form covers all self-organized processes endowed with endogenous intentionality. It is quite clear that economic systems belong to this second category since economic agents perform intentional tasks. On this basis, we can elaborate a little further and make an analytical partition between "common agreed social intentionality" stemming from the subsystems, e.g. agents, and "purely individualistic intentionality". This distinction presents a particular interest for economic analysis since it raises the old question of the origins of social (macroscopic) changes. As we know indeed, the history of economic throught shows that those two conjectures have been borrowed in turn. K. Arrow (1994) in a recent paper precisely stresses this distinction within the partisans of methodological individualism and seems to adopt, as Menger (1883) did, a reserved point of view: "some social changes occur conscientiously and for pragmatic reasons, while others are indeed unintended outcomes of individual actions. ... Particularly striking is the emergent nature of social phenomena, which may be very far from the motives of the individual interactions" (Arrow, 1994, p. 3). Between those two extreme cases we can, of course, envisage intermediate ones. In particular, some interest is here devoted to the case where individual intentionality is diffused due to non-linear feedbacks stemming from the economic system.

It now becomes clear that self-organization has been as Laslier and Laffond (1992) put it, an old flirting of economic analysis. And indeed Smith's "invisible hand" has been adequately, interpreted as a self-organization process (*cf.* e.g. J.P. Dupuy, 1983), as well as the **Industry and trade** motto: "The many in the one, the one in the many" and other methodological writings of A. Marshall (*cf.* e.g. Maricic, Ngo Mai, 1991; Foster, 1993; Hodgson, 1993). Hayekian "spontaneous order" or Schumpeterian "creative destruction" constitute also some quite evident instances of self-organizing processes (*cf.* e.g. Witt, 1992). Again, this is not surprising since problems of decentralised equilibrium models, as is well known, addressed such an issue but were not successful, if not tautological: for the ultimate argument about coordination rests upon an exogenous fiction (*cf.* e.g. Coricelli, Dosi, 1988; Kirman, 1989; Arrow, 1994).

As we shall see, self-organization is, in principle, able to reconsider this fundamental issue directly at the complex level of interactions among economic agents. Indeed most models in the related economic literature deal with strong forms of self-organization in order to modelize the emergence of relatively ordered paths out of partly unintentional outcomes of interactions between economic agents. The coordination issue is understood against the background of evolutionary mechanisms and leads to the notion of "viable coordination" (Witt, 1985) where agents try to keep within some viability bounds defined as the interplay of some processes of generation of variety and of selection (*cf.* e.g. Metcalfe, 1993; Witt, 1992*a*; Silverberg *et al.*, 1988).

It is quite clear that market issues constitute a privileged area of application of such mechanisms. In particular, different phenomena related to the labour market have been investigated (*cf.* e.g. Lesourne *et al.*, 1985, 1989; Lesourne, 1991, 1993; Laslier, Laffond, 1992) as well as innovation diffusion (*cf.* e.g. Silverberg (1988); Silverberg, Dosi, Orsenigo, 1988; Arthur, 1988, 1989), localization of activities (*cf.* e.g. Lesourne, 1993; Arthur, Krugman) or financial markets (Orléans, 1990, 1991). The evolutionary competitive dynamic as a self-organization process has also been generically studied (*cf.* e.g., Witt, 1985, 1993; Metcalfe, 1992, 1994). In most of those models, replicator dynamics and R.A. Fisher frequency-dependency equation stand for a useful modelling device (*cf.* e.g. Hofbauer, Sigmund, 1988).

It is whorthwhile to note that, according to our previous typology, some self-organization models have combined intentional individual behaviours with the emergence of social orders; in particular institutional issues have been dealt with in such a way. For instance the conditions of emergence and viability of a labour union from an intentional embryo have been worked out (*cf.* e.g. Lesourne, 1991, 1993). In the same line of ideas self-organization processes have been referred to in order to explain the emergence and permanence of established behaviour-guiding norms of whatever origin or nature (Coricelli *et al.*, 1991) –e.g. technological paradigm (Mosekilde Rasmussen, 1986) or the institutionalist instrumental valuation (Radzicki, 1990). In this latter case the coordination can emerge either from non-intentional outcomes of interacting individuals or from the interactions of some germs of collective intentions and individuals which lead to some behaviour-guiding norms due to some systemic effects.

On a more abstract level, we can note that the change of perspective implied by self-organization –to focus on interacting subsystems– is comparable with a change in generic methodological principles. The scope of this new appreciative paradigm, so to speak, can still be described through generic characteristics which stand over all self-organization economic processes. We shall now briefly comment some of those.

To begin with, let us note that there is a priori no incompatibility between self-organization and methodological individualism. The latter has been very recently both reasserted as a sacred tenet of economic analysis and minimized as the only explanatory principle of social variables (*cf.* e.g. Arrow, 1994). Applied to microeconomic problems –*i.e.* when sub-units are economic agents– self-organization meets some weak from of methodological individualism which departs from pure nominalism (*cf.* e.g. Brochier, 1994). In other words, far from denying any interest or reality to collective phenomena, self-organization tries to connect social variables to individual ones. Typically macroscopic structures are thought to be the product of micro-behaviours with non linear feedbacks which stem from the prevalent structures. Causalities between microeconomic and social variables are then not "one way thinking", as in a static world, but circular and complex. In particular, one is not led to a strict opposition between methodological individualism and holism. Individuals may be the unit of analysis but they are not conceived in isolation. Interactions among individuals can create collective rules which in turn, in a dynamic world influence further individual interactions. This way of thinking constrains the kind of rationality which is attached to the interacting units.

Self-organization models generally adopt some bounded forms of rationality; in particular they reject the standard intertemporal optimization framework (cf. e.g. Prigogine, 1993). Empirical studies could justify this quite non-orthodox conjecture but there is more. From an analytical point of view what seems unsuited in the standard maximizing rationality is the Robinson Crusoe preconception of economic agents. The level of the isolated individual seems to imply a different nature of rationality from the level of individuals inserted in a population. To focus on interactions means here more uncertainty and complexity; in a word, individuals are not alone in a static natural world which sends relevant economic signals, but face a complex environment. In such a dynamic world the most rational individual response might be to adopt bounded rationality, for heavy calculations are costly. The more complex the individual environment, the more routinized the behavioural response (cf. e.g. Heiner's competence-difficulty gap). This means that (i) individuals seek to control a very limited set of variables, (ii) the control process is approximative, quite simple and iteratively applied (*cf.* e.g. Nelson, Winter, 1982's "routines"; R. Day, 1986, 1993, 1994's Economizing behaviour; Leijonhufvud, 1993's algorithmic man; Arthur's "artificial man", 1993) and (iii) diversity in individual behaviours is allowed: some agents, instead of applying "routines", try to change their local economic environment in adopting innovative behaviours.

It is now essential to note that this conjecture must be associated with some interactions among agents: if bounded rationality is only defined at the level of the isolated agent self-organization processes might never appear. A simple form of interaction can be illustrated by imitative behaviours. This behaviour juxtaposed with positive feedbacks (e.g. Increasing return to adoption) has been widely used in economic self-organization models and largely contributes to explain emergent macroscopic structures (*cf.* e.g. Arthur, 1989; Dosi, Kaniovski, 1994).

To allow such procedural individual rationalities completely discards any reference to some representative agent. As is well known, this standard assumption leads to assume that choices of all agents (or category of) can be considered as the choices of one representative standard utility maximizing individual. These choices are then supposed to coincide with the aggregate choices of the heterogeneous individuals: it is the standard way to link microdiversity to macroregularities known as microfoundations of macroeconomics. It has been shown that this conjecture is no only unjustified but can also lead to wrong conclusions (*cf.* e.g. Kirman, 1989; Lippi, 1988). Self-organization paradigm is able to challenge this issue on the basis of individual stochastic interactions. As noted above to abandon strong methodological individualism makes it possible to focus on interacting agents endowed with procedural rationality. This framework –apart from the extreme

"common agreed social intentionality" self-organization processes– leads to a less rational macroeconomics since macroregularities stem from systemic effects, but in return can explain complex qualitative change phenomena as a result of some microdiversity (*cf.* e.g. Scheinkman, Woodford, 1994; M. Aoki, 1993; Weidlich, Braun, 1993).

Self-organization refers to the study of the global behaviour of complex dynamic systems as it results from the interaction of a large number of subsystems. Self-organization modelling implies at least two levels of observation:

– On the one hand, we have microscopic interactions implied by a very large number of individual decisions. It is then quite clear that the different sequences of events that may occur as outcomes of numerous individual interactions must be dealt with in a probabilistic form. While this description necessarily implies a loss of information about the exact situation of each subsystem, it remains that individuals constitute the unit of analysis; for their decisions determine the probability of occurrence of events. Of particular importance from this probabilistic description of microdiversity is the notion of stationary state of distribution of the stochastic process which represents the most probable outcomes of all possible sequences of events. This leads us to think about current events as generated by average behaviours. What can be perceived at the macroscopic level is typically the most probable event(s) which stem(s) from average behaviours.

– On the other hand, to the extent that the distribution of probability reached an equilibrium or stationary state, we observe some macroscopic regularities. This order may be represented in an appropriate deterministic way from a macroscopic point of view provided that the fluctuations of the underlying microeconomic processes –*i.e.* occurence of less probable events– cancel out. This deterministic representation of average micro behaviours is useful by itself and constitutes a second level of observation. This is of particular importance when one wants to investigate structural change phenomena. As is well known, this stylized fact received much attention recently with the emergence of non-linear dynamics (Batten, Casti, Johansson, 1987; Day Eliasson, 1986). But on the whole, it is still interpreted at a very aggregated level, almost remaining at a phenomenological level where micro decisions are out of the picture (for such an interpretation *cf.* e.g. Lordon, 1994). Beside this quite traditional point of view we think that self-organization might provide some probabilistic micro foundations which leads to look at structural change dynamics from a non phenomenological point of view. In particular, the role of non-average individual behaviours acquires, here, some importance in the explanation of structural change (see Ngo Mai, 1993).

This leads us to another important feature of the self-organization paradigm. Indeed such an analytical framework draws attention to pay attention to the stability issue. Until a recent period only stable equilibria were studied by scientists and by economists as well (Prigogine, 1993). With non linear dynamics new perspectives of economic instabilities were proposed. Indeed, multiple- equilibria or deterministic chaos provide an analytical framework where persistent endogeneous fluctuations may occur (*cf.* e.g. Grandmont, 1994). But this leads to a rather specific understanding of economic instabilities: either micro fluctuations, as defined above, are supposed to damp down and never emerged on a macroscale, only macro fluctuations are envisaged and/or the explicit reference to a representative agent erase any micro economic variety. In this field, self-organization proposes another type of explanation (Scheinkman, Woodford, 1994). The propagation effect of very small events due to non linear strongly localized interactions is introduced in the analysis of economic fluctuations. Under certain conditions, these small events can drive the average values to a new state or to put it differently, from the macro point of view, very small perturbations can lead to local instability of a prevalent state and drive it to another one (Nicolis, Prigogine, 1977).

In some cases, a convenient tool to modelize such a mechanism at the deterministic macro level is to test the structural stability of a prevalent state for very small perturbations. Some criteria to check the eventual spread of a small structural fluctuation are available (*cf.* e.g. Allen, 1976). This leads us to remark, that a special issue of the self-organization paradigm concerns the dialogue between diversity and stability. In the present state of the art this link does not seem quite clear yet, for under certain conditions diversity protects from fluctuations and leads to stability, while in certain cases diversity leads to local instability and is needed for evolution to occur (Allen, 1988; Kaufmann, 1993).

It is now well established that self-sustaining deterministic paths, either periodic or chaotic, can occur in well-formulated nonlinear models. This framework assumes that economies possess intrinsically unstable dynamics, which even in the absence of external shocks result in persistent fluctuations and growth. As Prigogine (1993) pointed out, "the fundamental dynamical level corresponds to unstable dynamical systems, which lead to time symmetry breaking. (...) The basic element for me is really the existence in nature of unstable dynamical systems. All other remarkable properties (bifurcations, chaos, the generation of information and time symmetry breaking) are in some sense consequences of the existence of unstable dynamical systems" (Prigogine, 1993, p. 2). From this standpoint, the study of the dynamics of industrial capitalism should not proceed as the solution of a given system subject to exogenous shocks. In line with Goodwin (1993) we consider that the economy has to be characterised by the fact that "it generates not only perpetual motion but one which exhibits continual alteration of its own structure in the pursuit of private profit" and that "It is ideally suited to self-organization in the specific sense that it consists of a great number of parts, all related, directly or indirectly". Nevertheless, most nonlinear models of growth and cycles focus on a set of deterministic equations which describe system behaviours over time. As noticed by Allen (1993) these equations only concern average behaviours. Now, "underneath the models or reduced descriptions, there will always be greater particularity and diversity of reality. (...) Real systems evolve, that is they add and substract mechanisms, components and interactions over time, whereas the deterministic model does not. Evolutionary change therefore must result from what has been removed in the reduction to the average description from the complete system. Determinism has been brought at the expense of structural change" (Allen, 1993, p. 101). In a series of recent studies it has been shown that when non average perturbations are reintroduced, there is an "evolutionary drive" that selects population with the ability to learn, rather than with optimal behaviour. This corresponds to the selection of "diversity creating mechanisms in the evolution". The master equation approach of birth and death processes is able to provide such a modelization (Haken, 1977; Allen, 1988; Orléan, 1990; Weidlich, Braun, 1992).

III. SELF-ORGANIZATION, CONFIDENCE AND CYCLICAL GROWTH: AN ILLUSTRATIVE MODEL

This section illustrates how the key concepts mentioned above as being central to the self-organization approach can be applied to built an adaptive growth model.

We consider an economy with two distinct types of agents. The first type of agents called workers are assumed to supply labour L_t^s and to finance their consumption out of their wages [2]. We assume, adopting a Kaldor-Kalecki perspective, that they have no opportunity to accumulate capital and organize production.

The second type of agents alone, called entrepreneurs, have the opportunity to accumulate and operate the production. In the remainder of this section, we shall therefore restrict our attention to the macroeconomic consequences of the behaviour of this second population of "active" agents. We denote by E the set of entrepreneurs e, $e = 1, 2, \ldots, n$. Each one uses L_t^e labour and capital K_t^e to produce the unique good, which can be either consumed or accumulated.

Assuming constant return to scale, we can specify the production possibilities by a factor price frontier, or a profit function:

$$r_t^e = R^e(w_t) \tag{1}$$

which gives the maximum possible return per unit of capital, if labour is purchasable at the real wage w_t. This standard function is non negative, continuous, non increasing and convex. Finally, the crucial assumption of this example is to suppose, in line with Woodford (1989), that for some reasons, for instance informational ones, no borrowing or lending is possible between agents of the two types, so that agents have to finance their spending out of their own income. It follows from this impossibility of external financing, that the net capital accumulated by an entrepreneur must equal his saving out of the returns of the capital used for production. Given this assumption, the investment decision of agent e is:

$$I_t^e = \sigma_e R^e = \sigma_e \Pi_t^e \tag{2}$$

where $0 < \sigma_e < 1$ refers to the propensity to save out of profits. Hence, the evolution of the capital stock of this agent is given by:

$$K_{t+1}^e = (1 - \delta) K_t^e + I_t^e \tag{3}$$

where, $0 \leq \delta \leq 1$ is the depreciation rate.

We then introduce characteristics, which are crucial for the dynamics of the model: the state of confidence or animal spirits of entrepreneurs. Intuitively, it is rather obvious that the formation of this state of confidence is of a cooperative nature: the formation of an individual's opinion is influenced by the presence of groups of agents with the same or opposite opinion. In order to obtain a tractable model, we consider the simplest case of two opinions, a pessimistic one and an optimistic one, denoted by minus and plus.

Let n_- and n_+, with $n_- + n_+ = n$, be the numbers of entrepreneurs with the corresponding opinions. Denote respectively by E^- and E^+ the sets of n_- pessimistic and n_+ optimistic agents. Consequently we can characterize the investment decisions as follows:

$$\begin{cases} I_t^e = 0 & \text{for } e \in E^- \\ I_t^e = \sigma_e \Pi_t^e & \text{for } e \in E^+ \end{cases} \tag{4}$$

Next, we assume that each individual may change his state of confidence over time during the economic process.

The transition probability per unit of time, for a change of opinion of an individual from state + to − and vice versa are $p_{+-}(n_+, n_-)$ and $p_{-+}(n_+, n_-)$. In line with Haken (1977) and Weildlich and Braun (1993), the probabilities can be specified as follows:

$$p_{+-}(n_+, n_-) = v \exp\left\{\frac{-(\alpha x + \beta)}{\theta}\right\} \tag{5.a}$$

$$p_{-+}(n_+, n_-) = v \exp\left\{\frac{(\alpha x + \beta)}{\theta}\right\} \tag{5.b}$$

where, $x = \frac{n_+ - n_-}{n}$, θ refers a collective "climate" which facilitates the change of opinion or makes it more difficult, β is a positive preference parameter meaning that + is preferred to −. One can think of external informational influences on individuals. Finally α is a measure of the strength of the cooperative effect x. It is assumed that the rate of change of the confidence of an individual is enhanced by the group of individual with the opposite opinion. Let us define the probability distribution function $P(n_+, n_-, t)$ of finding n_+ optimistic agents and n_- pessimistic ones at time t. Thus, one may easily build a master equation which describes the motion of $P(n_+, n_-, t)$ over time. We get:

$$\begin{aligned} \frac{dP(n_+, n_-, t)}{dt} &= (n_+ + 1)\, p_{+-}(n_+ + 1, n_- - 1)\, P(n_+ + 1, n_- - 1, t) \\ &\quad + (n_- + 1)\, p_{-+}(n_+ - 1, n_- + 1)\, P(n_+ - 1, n_- + 1, t) \\ &\quad - \{n_+ p_{+-}(n_+, n_-) + n_- p_{-+}(n_+, n_-)\}\, P(n_+, n_-, t) \end{aligned} \tag{6}$$

Hence, while the probabilities $p_{+-}(q)$ and $p_{-+}(q)$ encapsulate microdiversity, this probability distribution $P(n_+, n_-, t)$, refers to the state of confidence at the macro-level. From this point of view, let us consider the aggregated values $K_t = \sum_e K_t^e$ and $I_t = \sum_e I_t^e$. We can now obtain the capital accumulation relation at the macro-level:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{7}$$

and the actual growth rate of the economy g is well-defined by:

$$g = \frac{I_t}{K_t} - \delta \tag{8}$$

The dynamics of this growth rate is to be explained by two elements, a stochastic one and a deterministic one.

On the one hand, it is clear that this growth rate is not constant; it fluctuates stochastically since the structure of E evolves over time according to (6). Hence, it appears that probabilistic individual decisions entail, via the modifications of the state of confidence, a fluctuating change of this macrovariable.

On the over hand, one may contend that the global state of confidence is not so unstable and that its fluctuations cannot adequately explain the instability of growth. In fact, the idea of macroeconomic order or regularity is not totally absent. As we know, mean values $\overline{n_+}$ and $\overline{n_-}$ can easily be derived from the master equation (*cf.* Weildlich and Braun, 1993). Let us assume that equation (6) admits always at least one stationary solution [3] $P^0(n_+^*, n_-^*, t) = 0$. For these values the structure of E is stationary. Consequently the law of motion of the capital stock depends solely on the evolution of profits and is deterministic.

If we suppose for instance that the real wage rate w_t balances the labour market [4], the evolution of the capital stock, and hence of the growth rate g, is given by a nonlinear difference equation:

(9) $$K_{t+1} = G(K_t)$$

Let us assume for instance that labour supply and demand functions are respectively given as:

(10) $$L^s = w^b, \quad \text{with} \quad 0 < b \leq 1,$$
$$L^d = (a_0/a_1)\,K, \quad \text{with} \quad 0 < a_0 < 1,\ 0 < a_1 < 1$$

Hence the wage rate writes as:

(11) $$w_t = V(K_t) = ((a_0/a_1)\,K_t)^{1/b}$$

The wage curve is given as

(12) $$r_t = (w_t + d)^{-1}$$

Hence, we get:

(13) $$K_{t+1} = (1-\delta)\,K_t + \sigma\,K_t\,(d + ((a_0/a_1)\,K)^{1/b})^{-1} = G(K_t)$$

This is a particular case of the discrete one dimensional dynamical systems analysed by Day (1982, 1993); Grandmont (1985) or Woodford (1989), in which complex deterministic dynamics can easily occur.

Consideration of whether or not self-perpetuating growth cycles are possible in such an economy requires investigation of the map G defined as the right hand side of (9). Under the assumptions made above, we know that G has a unique fixed point $K^* > 0$. Local stability of this unique positive steady-state capital stock depends on the elasticities of demand for and supply of labor. (See Woodford, 1989, p. 316.) The intuition is that if both labor demand and labor supply are relatively inelastic, the steady state is unstable and complex dynamics can occur. A small increase in the level of K^* results in a large one in the real wage rate, and hence induces a large fall of the profit rate. Hence total returns on capital can decrease as K^* increases. G has a negative slope at the steady state.

However, the main idea is that such a dynamics only refers to average or stationary micro-behaviours. When non average behaviours are reintroduced stochastic fluctuations driven by (6) reappear and the deterministic relations (9) (13) are discarded.

Now, if we consider a change driven by "non average behaviours" in the probability distribution underlying the stochastic process (6), new stationary or mean values can be defined. Hence, a new function $G_1(K_t)$ can be built which replaces the initial one $G_0(K_t)$. The process continues as long as there are modifications in average behaviors. Hence, in the long run, we can get a multi-phase dynamical process (Day, 1993).

We get for instance the following graph for two changes in mean values:

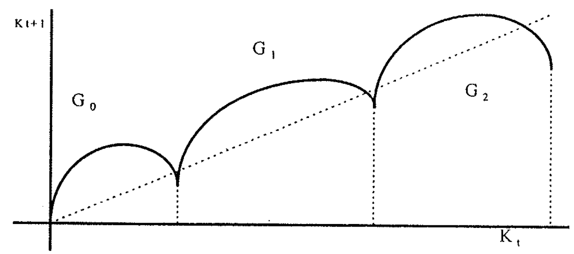

Figure 1.

A numerical simulation of the model illustres the fluctuating path of the growth rate [5] induced by deterministic and stochastic components in this process.

In figure 2a the simulated trajectory of the growth rate g_t is shown. In figure 2b, an empirical picture of the real growth rate for the US economy over the period 1946-1987 is shown.

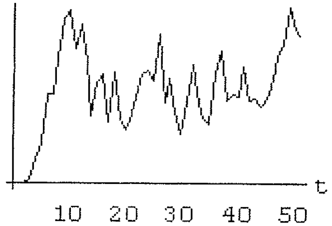

Figure 2a.

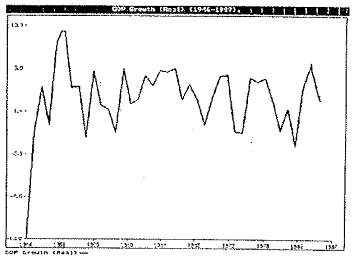

Figure 2b.

To quote Day (1993), "not too much should be made of such a casual comparison". However, the qualitative similarity in the existence of fluctuating growth path is quite impressive.

Hence, the heuristic model proposed above, although quite simple, illustrate self-organization advances in the field of cyclical growth analysis. Deterministic and stochastic approaches in economic theory generally represent conflicting ideas of endogenous and exogenous mechanisms of dynamical processes, yet they simply become complementary tools in an emerging paradigm in evolutionary economics.

Notes and references

1. *Cf.* Silverberg (1988) for a survey of the different mathematical tools used by the evolutionary analysis of technical change and Lesourne (1989) for a general state of the art specifically devoted to microeconomic issues.

2. For simplicity's sake we assume that workers supply a homogeneous quantity of labor.

3. Such a solution always exists when the transition probabilities are independent of time. Moreover, this solution is unique, provided the graph of the master equation is connected. (Haken, 1983, p. 97).

4. Let us suppose for instance that labour demand is $L_t^d = a_0/a_1 \, K_{t+1}$ where a_0 and a_1 stand for labor and good coefficients. Then the equilibrium wage rate os given by $w_t = V(K_t)$. (*Cf.* Woodford, 1989.)

5. Simulation done using Mathematica, with $\delta = 0.5$, $\sigma = 0.5$, $a_1 = 0.7$, $a_0 = 0.07$, $d = -5$, $b = 1$, $K_0 = 25$, $n = 30$, $n_0^+ = 15$, $n_0^- = 15$, $t = 50$.

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