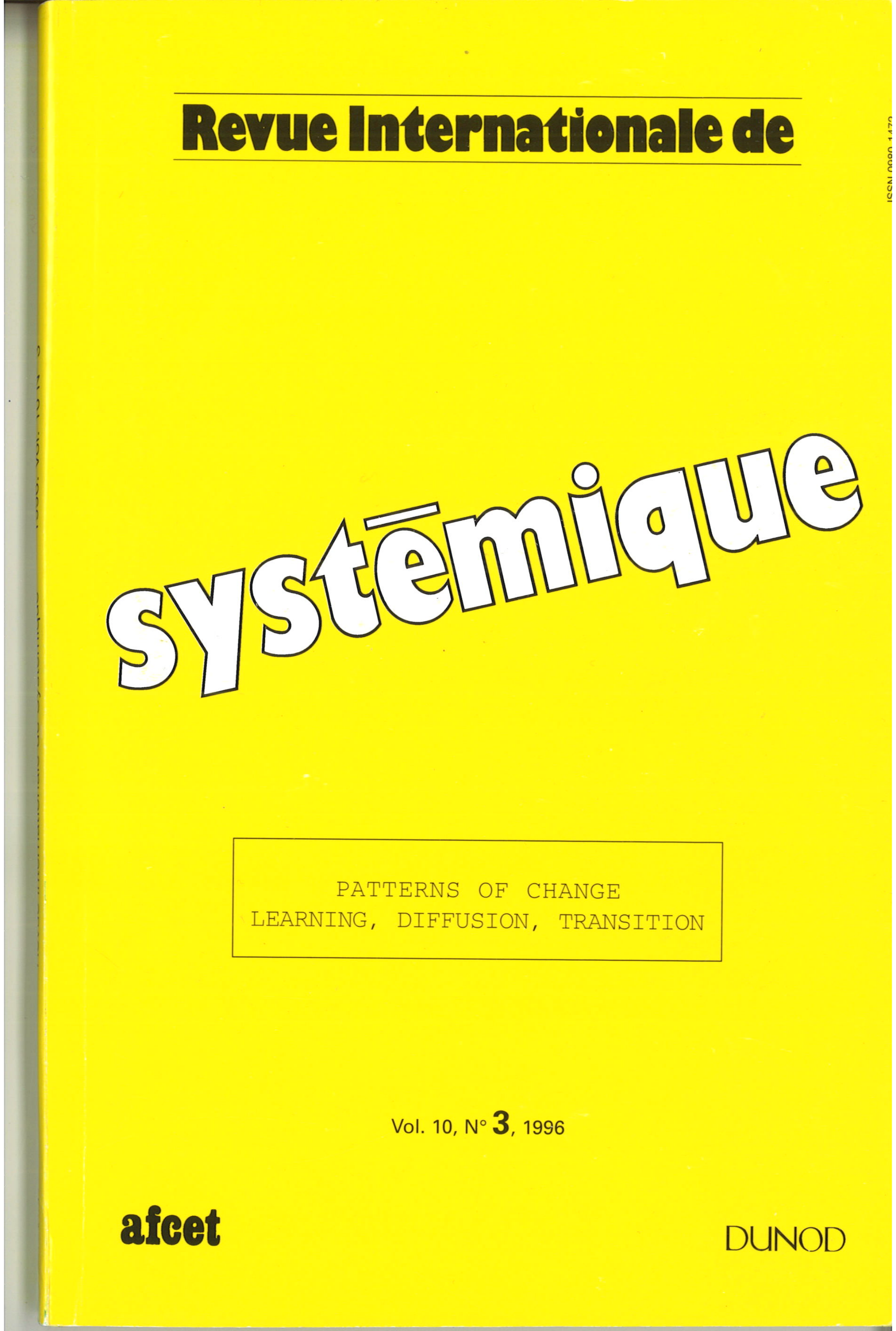

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INVENTIVE INVESTMENT AND THE DIFFUSION OF TECHNOLOGY*

Willi SEMMLER [1] and Alfred GREINER [2]

Abstract
In the paper we present a model that integrates evolutionary as well optimizing approaches to technical change. The innovation and diffusion of new techniques take place while competing technologies exist: an old technology operated by existing firms and a new technology introduced by innovating firms. The two types of firms dynamically interact. The firms that fully optimize when undertaking inventive investments maximize their returns on innovations. The other type of firms behaves purely adaptively and gradually adjusts as postulated in the evolutionary approach. The interaction of the two types of firms can produce multiple equilibria, path dependence, diffusion processes with varying number of firms and the existence of a diversity of techniques. Empirical evidence from diffusion studies is employed in order to obtain realistic parameters and stylized facts. The sensitivity of the market shares of firms is studied with respect to changing discount rates, elasticity of demand, affecting the markup of the innovating firms and the diffusion speed of innovations. In the present paper such results are obtained in a deterministic setting. Extensions to a stochastic environment are discussed in Semmler (1994).

Résumé
Dans cet article, nous présentons un modèle qui intègre des éléments évolutionnistes et des éléments optimisants en considération du changement technique. L'innovation et la diffusion d'une nouvelle technique a lieu tandis que des technologies en concurrence coexistent : une veille technologique opérée par des entreprises établies et une technologie nouvelle introduites par des entreprises innovantes. Les deux

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types d'entreprise disposent d'interactions dynamiques. Les entreprises qui suivent une stratégie optimisante en faisant des investissements innovants maximisent leur revenu résultant des innovations. L'autre type d'entreprise se conduit de façon purement adaptative et s'adapte graduellement, conformément à la théorie évolutionniste. L'interaction des deux types d'entreprises peut produire des équilibres multiples, la dépendance du passage, des processus de diffusion avec un nombre variable d'entreprises et l'existence d'une diversité de techniques. Le caractère empirique évident de la diffusion des études est employé pour obtenir des paramètres réalistes et des faits stylisés. La sensibilité de la portion du marché des entreprises est étudiée quant aux variations du taux d'escompte, à l'élasticité de la demande qui influencent l'augmentation des entreprises faisant des innovations et quant à la vitesse de diffusion. Dans le présent article, de tels résultats sont obtenus dans un environnement déterministe. Des extensions à un environnement stochastique sont discutées dans Semmler (1994).

I. INTRODUCTION

This paper analyses the process of technological change under the condition of competing technologies where one group of firms actively innovates and the other group, operating existing techniques, passively responds to changes in the technological environment. Innovating firms commit resources and undertake inventive investment expecting a return from it. They are postulated to optimize by computing the net present value of their revenue from the innovation. While the active firms aim at harvesting a technological rent the group of passive firms do not optimize though, under competitive pressure, they may learn to improve their efficiency. Due to several kinds of interactions among the two groups of firms the model allows for multiple equilibria and trajectories unknown in models with one state variable only.

To be more precise let the active group of firms maximize the following net present value

$$\left.\begin{aligned} V_{\max} &= \int_0^\infty e^{-rt}\, g(x_2,\, u)\, dt; \qquad u \in \Omega_+ \\ s.t.\ \dot{x} &= F(x,\, u) \end{aligned}\right\} \tag{1}$$

where $g(x_2, u)$ is the return function, u a control variable, x_2 the number of innovating firms, x_1 the number of firms operating the existing technique and $F(x, u)$ the law of motion in the state space, described by an ordinary differential equation system, and $r < \infty$, the discount rate. In contrast to the above intertemporal (dynamic) optimization problem there exists, as shown in Sieveking and Semmler (1996), a static optimization problem with zero time horizon as the limit of (1) when the discount rate r tends to infinity. The static optimization problem reads

$$\left.\begin{aligned} &\max_u g(x_2,\, u); \qquad u \in \Omega_+ \\ &s.t.\ \dot{x} = F(x,\, u) \end{aligned}\right\} \tag{2}$$

With regard to the optimal trajectories of the state variables the study of the dynamics of (2) allows to conclude back to the dynamics of (1) for large enough discount rates (*cf.* Sieveking and Semmler, 1996). A dynamic programming algorithm, written for studying the intertemporal version (1), permits to compute the optimal trajectories for any discount rate. It becomes, therefore, interesting to study the market share of techniques of firms when the discount rate, the elasticity of demand, affecting the net revenue, or the diffusion speed of new technologies vary. We will particularly focus on the diffusion speed.

Frequently, economic theory assumes that information about new technologies and products is a kind of public good that can costlessly and timelessly be acquired. In our model information on the technologies has to be obtained at a certain cost [1]. Resources have to be spent to attain the knowledge on new technologies (either by creating or purchasing it) [2]. For the new innovation the resource cost is to be compared to the gains –in general, the discounted present value of profit flows generated by the innovation [3]. Accordingly, best-practice techniques are not always instantaneously adopted and implemented by all firms. The diffusion process of new technologies, competing with existing ones, can be rather complex. The approach put forward here relates to three types of recent literature.

A first strand of relevant research was initiated by Arrow (1962) and has subsequently been extended by Dasgupta and Stiglitz (1980*a*, 1980*b*), and Loury (1979), among others. Here only one type of firms is considered: the strategically operating (optimizing) firms. New technologies are introduced at a certain resource cost but the innovating firms do not interact with firms operating existing techniques. The effort made by firms to innovate (usually perceived as R and D cost) is contrasted with a revenue from the innovation –the revenue determined by a downward sloping demand curve [4]. In some of the papers emphasis is given to uncertainty and risk involved in the dynamics of innovation and diffusion. It is stressed that technological and market uncertainties arise in the relationship of research effort and the gains that can be captured from introducing the innovation. This, in particular, is due to the interdependence of firms' decisions and possible rivals' reactions when new technologies are implemented.

A second line of reseach is present in the work by Arthur (1988, 1989) who has recently rekindled the debate on innovation and diffusion. He stresses the fact that firms' new techniques have to compete with existing old techniques. In the adoption process of new technologies probabilistic events, returns to scale and positive feedbacks play an important role (*cf.* Arthur *et al.*, 1986) [5]. Due to a positive feedback of stocks and flows in the innovation process, an early implementation of a new technology can usually enjoy increasing returns to adoption resulting from learning by using, network externalities, scale economies in production, and increasing returns to information and skills. Insignificant early events may thus give rise to an initial advantage in adoption, and with adoption the advantage will be increased further. The new technology then may finally "corner the market".

A third type of literature is known as the evolutionary approach to technical change. It revitalizes in particular the Schumpeterian theory of technical change [6] by borrowing from recent advances in mathematical biology to stylize the processes of innovation and diffusion. This direction, however, expresses some skepticism concerning the postulate of full-fledged optimizing behavior of economic agents. Rather it argues that a firm's behavior in a changing environment is guided by limited information and bounded rationality. The informational requirements and computational complexity involved in fully rational behavior would give rise to an unlimited cost of optimizing since, rigorously speaking, the assumption of rational behavior implies that the cost of such optimizing behavior should be part of the model's solution. Because it seems unreasonable to expect a full account of the cost of optimizing in a complex and uncertain environment, some rule other than optimization, such as satisficing or stopping rules, should be considered the foundation of the firm's behavior (*cf.* Conlisk, 1989). Limited information, risk, and computational complexity would consequently give rise to uncertainty concerning which of the competing techniques will be implemented [7].

Most empirical studies on the diffusion of technologies have demonstrated that new products and processes are usually changed during the diffusion process. As a consequence, the early models of diffusion which assumed an unchanged product diffusion through an unchanged environment have been largely displaced (see e.g. Stoneman, 1983; Mansfield, 1989; Nakicenovic and Gruebler, 1991, or Midgley *et al.*, 1992). So, the automobile or the computer for example have been modified and improved during their diffusion. However, if the product or process is improved during the diffusion process these improvements are often only gradual and to be treated as incremental innovations rather than radical. But there are also products which change little if at all. Examples can be found especially in the drug industry or food industry which did not show any or at most minor modifications.

Another result of the empirical studies is that the rates of diffusion vary to a great degree. This variation can be observed both for different products and for different countries (*cf.* Rosenberg, 1976; Ray, 1984; Mansfield, 1989, or Dosi, 1991). It was also demonstrated that the productivity gains associated with the diffusion vary strongly (see e.g. Antonelli, 1986, 1993, for the telecommunications and telematic service sector).

Further, the number of firms does not stay constant during the diffusion process. Jovanovic and MacDonald (1994) showed that the number of firms first rises and then falls as an industry evolves. The reason is that at the beginning innovation opportunities attract firms, but the failure to innovate in later stages finally leads to a decline in the number of firms.

Our paper incorporates some features of the above strands of inquiries. The analysis will, however, be restricted to studying models of deterministic dynamics [8].

We will construct a model of innovation and diffusion whose dynamics is reminiscent of evolutionary models. As other recent papers, we will refer to models of interacting populations which develop three essential types of interactions – cooperative, predator-prey, and competitive (*cf.* Hofbauer and Sigmund, 1984). Two variations of our basic approach will be explored. In one variation, presented in section II, firms behave with a zero time horizon acquiring a new technology. The model will be of type (2) above. In this version rationality is somewhat "bounded" as is more characteristic of the third approach discussed above. In a second variant of our model, discussed in section III, the innovating firms optimize intertemporally as characteristic for the system (1). A dynamic programming algorithm, as developed in Semmler (1995), will be employed to study the trajectories of the market shares of the different types of firms.

Generally, however, with two state variables of our dynamic system, optimizing behaviors as formulated in (2) and (1) admit multiple equilibria, path-dependence (in the sense of David, 1990, and Arthur, 1989), complicated diffusion processes, and coexistence of different techniques.

II. INVENTIVE INVESTMENT AND STATIC OPTIMIZATION

As mentioned previously, we classify firms into two classes: firms that produce with the existing technique, behaving passively and firms behaving actively, *i.e.* optimize when investing in new technologies [9].

Optimizing, in the case of active firms, means that there is a full accounting of the resources for obtaining new technological knowledge. Firms expect a net revenue from the inventive investment. We want to posit that the new technology will be either invented or acquired at a certain cost – an innovation or adoption cost. The total cost for operating the new technology is assumed to be dependent on the effort spent to obtain the new technology (independent of the number of firms) and on a cost proportional to the number of firms operating it. We do, however, not presume that perfectly competitive conditions hold so that the innovation rent is instantly dissipating. The new technology is employed monopolistically (through joint profit maximization of innovating firms) [10]. When the active firms innovate and compare the cost of innovation with the gains from it – which in general will be equivalent to the present value of future profit flows – the firms are admitted to exhibit positive or negative profits. Along the lines of Smith (1968) and Berck (1979) entry into the group of innovating firms from outside the industry is encouraged when profits are positive and exit occurs with negative profits. Thus, the process of rent dissipation is assumed to be rather slow [11].

In addition, we presume three types of interaction effects [12] between the innovating firms and firms operating the exiting technology: a predator-prey relation between the new and the old firms; a cooperative effect; and a competition (or crowding) effect. The predator-prey relation stems from the effect that we also allow existing firms to switch over to the new technology and thus new firms grow at the expense of the old firms. The crowding effect results from a price or mark-up squeeze due to the introduction of a new technique. We will use an inverse demand function to specify this effect [13]. A cooperative effect (spillover or learning effect) will bound the number of old firms away from zero, so that complete extinction is avoided.

With a costly new technique, the innovating firms most likely will have an unprofitable period when the new technique is introduced. They then face a period when they can capture a technological rent (transient rent), and finally lose their rent due to subsequent crowding effect as a result of an increase in their numbers [14].

In the present version of the model optimizing takes place with a short time horizon, or rather the time horizon approaches zero. The optimization problem that arises from it is of type (2) above where the discount rate approaches infinity [15]. Economically, the meaning of this is that firms heavily discount the future because of fierce competition, informational constraints, uncertainty and high risk about next period's revenue. For a theory of technical change, where one usually postulates a strong interdependence of firms' decisions, the static optimization approach where $r \to \infty$ appears to be an adequate starting point.

The optimizing version (with time horizon of firms approaching zero) reads as follows :

$$\max_u g(x_2, u);\ u \in \Omega_+$$

with $g(x_2, u) = \mu(x_2, u)\, x_2 u - cu - c_0 x_2$, $\mu = \alpha/(\phi + x_2 u)$, subject to

$$\dot{x}_1 = k - a(x_1, x_2)\, x_2 + \gamma\beta x_2 - e(\phi + x_2 u)\, x_1, \tag{3}$$

$$\dot{x}_2 = x_2(a(x_1, x_2) + vg(x_2, u) - \beta), \tag{4}$$

where k is a constant, x_1 is the number of firms operating the existing technique, x_2 the number of innovating firms, and u a control variable from the control space Ω_+. Note that we require the control u to be non-negative. The control variable u indicates the level of effort spent to invent or acquire the new technology [16]. The cost per unit of effort is denoted by c. The cost cu is independent of the number of firms [17] and there is a cost proportional to the number of firms, $c_0 x_2$. Thus, $cu + c_0 x_2$ is the amount of resources that innovating firms have to devote to the innovation. The total cost of innovation can be conceived as the entry-cost to the new technology (for the innovating firms as a group). The term $\mu(\cdot)$ is the (net) price received for the product produced by the new technology, with $\mu(\cdot)\, x_2 u$ the net revenue [18].

When firms maximize a technological rent $g(\cdot)$, facing a revenue $\mu(\cdot)\, x_2 u$ and the cost $cu + c_0 x_2$, a positive rent will increase their number. In (4), the term $vg(\cdot)$, with v a constant, means that there is entry of firms from outside the industry proportional to the excess profit of the innovating firms [19]. Moreover, in (4) $\beta > 0$.

The term $a(\cdot)\, x_2$ which is specified as $a x_1 x_2 x_2$, a being a constant, represents the predator-prey interaction where the adoption of the new technique is supposed to take place proportionally to the product of x_1 and $x_2 x_2$ (a common assumption for the spread of information in sales-advertising models, *cf.* Feichtinger, 1992) [20]. This implies that as the number of firms applying the new technique grows, so does accessibility of that technique for old firms. This way the rate of decrease of old firms in (3) is translated into an equivalent rate of increase of new firms in (4).

The term $\gamma\beta x_2$ in (3) reflects the cooperative effect of x_2 on x_1 [learning by old firms [21] and in addition, firms may commence again with the old technology after being eliminated by the dynamics (4)]. The last term

$e(\phi + x_2 u) x_1$ in (3) is the crowding effect for x_1 which is specified as $(e/\mu) x_1$ with e a parameter and μ the price determination from an inverse demand function which also appears in (4).

For simplicity, taking $\alpha = 1$ we have $\mu = 1/(\phi + x_2 u)$. The optimal u^* can be derived by taking for $g(x_2, u)$ the derivative

$$\frac{\partial (x_2 u/(\phi + x_2 u) - cu - c_0 x_2)}{\partial u} = 0$$

and solving for the optimal u^*, which depends on the parameters of the return function $g(\cdot)$ and x_2 [22]. In general, we get:

$$u^* = (\sqrt{\phi x_2/c} - \phi)/x_2. \tag{5}$$

Furthermore, for our parameter constellations, the following properties for the return function hold [23]:

$$\begin{aligned} g(x_2, u^*) &< 0, && \text{for} \quad x_2 < x_{2\,\min}; \\ g(x_2, u^*) &> 0, && \text{for} \quad x_{2\,\min} < x_2 < x_{2\,\max}; \\ g(x_2, u^*) &< 0, && \text{for} \quad x_2 > x_{2\,\max}. \end{aligned}$$

Moreover, as numerically explored, for our parameters the return function $g(\cdot)$ is concave in x_2 within the above boundaries [24]. The boundaries $x_{2\,\min}$, $x_{2\,\max}$ are derived from the following considerations.

PROPOSITION 1 : *There is a minimum number of firms $x_{2\,\min}$ in order to obtain a nonnegative profit* ($g(\cdot) = 0$).

Note that $\mu(x_2, u) = 1/(\phi + x_2, u)$ is falling in x_2 and u and that u^* depends on x_2 [*cf.* (5)]. Then the nonnegative profit condition above means that there is a $x_{2\,\min}$ such that

$$(1/(\phi + x_{2\,\min} u^*)) x_{2\,\min} u^* - cu^* - c_0 x_{2\,\min} = 0. \tag{6}$$

The proposition is demonstrated using our numerical parameters, *cf.* Appendix 1. Note, however, that there is no other u that maximizes $g(\cdot)$ and that u cannot be set to zero even if $g(\cdot) < 0$. The latter part of the statement comes from the fact that there is a cost independent of $u\,(c_0 x_2)$ which forces firms to operate the new techniques even if they will have losses. With an optimal u losses are, however, minimized (*cf.* Appendix 2 for a numerical example).

PROPOSITION 2 : *There exists a number of firms $x_{2\,\min} < x_2 < x_{2\,\max}$ for which an optimal u^* generates a maximum of profit. Substitution of u^* from (5) into $g(x_2, u)$ results in $g(x_2, u^*) > 0$ for the interval $x_{2\,\min} < x_2 < x_{2\,\max}$* (cf. *Appendix 2 for the numerical computation).*

PROPOSITION 3 : *There is a $x_2 > x_{2\,\max}$ for which, with u^*, the profit becomes negative. Thus, there is a decline of the (net) revenue with increasing x_2 above a certain critical $x_{2\,\max}$* (cf. *appendix 2 for the numerical calculation of $x_2 > x_{2\,\max}$).*

There are multiple equilibria of the system (3)-(4) with an optimal control u^* as defined in (5). In the $\mathbb{R}^2_+$ we have found at most four equilibria, depending on the diffusion speed a. For $a = 0.01$ the equilibria are given by: [25]

$$\begin{aligned} &(E1): x_1^* = 1.2,\ x_2^* = 17.1; && (E2): x_1^* = 20,\ x_2^* = 0; \\ &(E3): x_1^* = 31.1,\ x_2^* = 0.2; && (E4): x_1^* = 12.7,\ x_2^* = 1.2; \end{aligned}$$

For our parameter constellations there are three economically meaningful equilibria of the optimally controlled system (3)-(4). These are (E1), (E2) and (E4). (E3) implies a negative control u^* for our parameter constellations and is therefore not viable. The equilibrium (E2) is obtained by setting the control $u^* = 0$ when u^* passes through zero from above. Thus (E2) is a viable equilibrium. (E2) can easily be computed by setting $x_2 = 0$ in (4), employing $u^* = 0$ and solving for x_1^*. The other equilibria were computed numerically. The computation of the Jacobian of the optimally controlled system (3)-(4) shows that

$$\begin{aligned} J_{11} &= -a(x_2^*)^2 - e(\phi + x_2^* u^*); \\ J_{12} &= -2\,a x_1^* x_2^* + \gamma\beta - e x_1^* (u^* + x_2^* \partial u^*/\partial x_2); \\ J_{21} &= a(x_2^*)^2; \\ J_{22} &= 2\,a x_1^* x_2^* - \beta + v\,\partial g(x_2^*, u^*)/\partial x_2; \end{aligned}$$

For our parameter values we obtained $\partial u^*/\partial x_2 < 0$ and $\partial v g(x_2, u)/\partial x_2 > 0$ for $x_2 \leq 7$ and $\partial v g(x_2, u)/\partial x_2 < 0$ for $x_2 > 7$ in numerical evaluations.

For the study of the stability properties of the equilibria (E1), (E2), (E4) we employed the diffusion speed, represented by a, as a bifurcation parameter [26].

A detailed study of the behavior of the trajectories in the vicinity of the equilibria (E1), (E2), (E4) was conducted by computer simulations. The equilibrium (E4) proved to be unstable for small as well as for large a (and for a variety of initial conditions). From the large number of computer simulations for the other equilibria we select only few interesting cases [27].

In the first type of simulation we keep the initial conditions constant. For our parameter constellations and a large enough a, *i.e.* $a \geq 0.04$, and the chosen initial conditions, (E1) becomes an attractor. However, with a minute a, *i.e.* $a = 0.01$, and same initial conditions, (E2) turns out to be an attractor [28]. These two cases are demonstrated in figures 1 and 2.

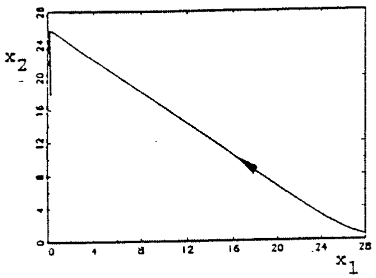

Figure 1. $a = 0.04$, $x_1 = 28$, $x_2 = 0.6$

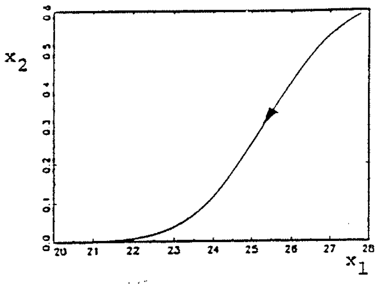

Figure 2. $a = 0.01$, $x_1 = 28$, $x_2 = 0.6$

On the other hand the fact that the dynamics depend on the initial conditions is illustrated by the next two figures where we take the parameter a constant. As figures 3 and 4 show, for initial conditions $x_1 = 33$, $x_2 = 1$, (E1) is an attractor; for $x_1 = 33$, $x_2 = 0.6$, and the same diffusion speed a, (E2) is the attractor.

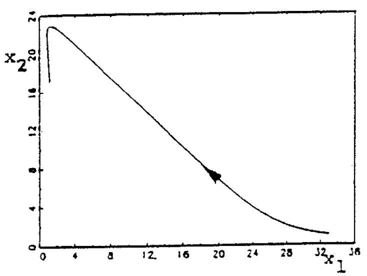

Figure 3. $a = 0.01$, $x_1 = 33$, $x_2 = 1$

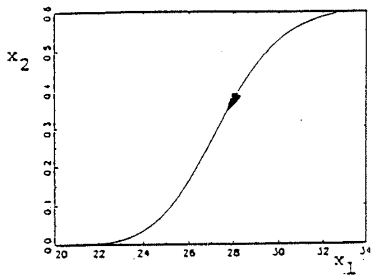

Figure 4. $a = 0.01$, $x_1 = 33$, $x_2 = 0.6$

Figures 1 and 3, with the trajectory converging toward the high level equilibrium (E1), are quite appealing. The trajectories are comparable to the outcome of models with one state variable only (the logistic approach). With two types of firms, as in our model, the new and the old technology can co-exist. One also can observe that the market share of the new technology

first rises and then falls. Figures 2 and 4, on the other hand, where the low level equilibrium (E2) is an attractor, demonstrate that innovating firms do not necessarily succeed in increasing their market shares when their number is too small. The market share of the new technology can decrease to zero (even with a control $u^* > 0$ for some regions of the trajectories).

The above examples of our computer studies – for the limit case of $r \to \infty$ – illustrate the possibly quite complex outcomes of innovation and diffusion processes when optimal behavior and two types of (interacting) firms are allowed for. The studies indicate that a slight change of the diffusion speed expressed by the parameter a, or initial conditions, can give rise to a quite dissimilar behavior of the trajectories. Depending on the parameter a and/or the initial conditions the trajectories can move to low or high level equilibria (the trajectories display the market share of the different technologies). In general, the change of the trajectories after altering the parameter a and/or initial conditions stems either from a change of the stability properties of the equilibria or from the change of the isoclines of the equilibria [29].

III. INVENTIVE INVESTMENT AND DYNAMIC OPTIMIZATION

Next, we consider the infinite horizon model (1) where the net revenue of future periods are discounted with a discount rate $r < \infty$. In general, trajectories obtained for an infinite discount rate should be reproducible for a finite, but large, discount rate r.

For our model specifications the intertemporal optimization version with the finite discount rate reads as follows

$$V_{\max} = \int_0^{\infty} e^{-rt} (x_2, u)\, dt; \qquad u \in \Omega_+$$

with $g(x_2, u) = \mu(x_2, u)\, x_2\, u - cu - c_0\, x_2$, $\mu = 1/(\phi + x_2\, u)$, subject to

$$\dot{x}_1 = k - a x_1\, x_2^2 + \gamma\beta\, x_2 - x_1\, e/\mu, \tag{7}$$

$$\dot{x}_2 = x_2\, (a x_1\, x_2 + v g\, (x_2, u) - \beta), \tag{8}$$

For the numerical computation of the value function and the optimal trajectories a discrete time dynamic programming version of the above model was employed with step size h. One would expect the following relation to hold between the discrete time and continuous time version of the value function

$$V_{\max} = \lim_{(h,\, g,\, \varepsilon) \to (0,\, 0,\, 0)} W^{h,\, g,\, \varepsilon}$$

with h the step size, g the grid size and ε the interval at which the discrete control variable u is applied. Furthermore, the discrete time dynamic programming form is [30]

$$W = \max_{u} \left\{ hg\left(x_{2t},\, u\right) + e^{-rh}\, W\left(x_t + hf\left(x_t,\, u\right)\right) \right\} \tag{9}$$

The discrete time dynamic system, with the optimal control u^*, in compact form reads:

$$x_{t+h} = x_t + hf\left(x_t,\, u^*\right) \tag{10}$$

The value function (9) can then be iteratively computed.

According to Sieveking and Semmler (1994), the dynamic results stemming from $r \to \infty$ should also hold for $r < \infty$ with a large r. Therefore, for (9), we expect trajectories that are closely related to trajectories resulting from (3)-(4). Concerning the discount rate, we want to add that it can be conceived of as composed of two factors: an interest rate and risk factor – the latter expressing the risk or uncertainty of future periods' profit flows [31]. The parameters employed in the computation are reported in appendix 1. Using a diffusion speed $a = 0.01$ and a discount rate $r = 2$ for this example the value function and the optimal trajectories are computed.

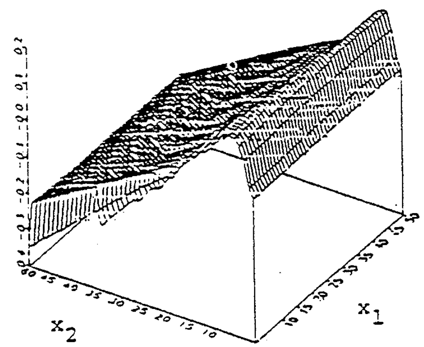

Figure 5. The value function $(r = 2)$.

Figure 5 demonstrates the value function as computed from (9) with $r = 2$. The axes x_1 and x_2 represent the number of firms employing the old and new technologies and the vertical coordinate shows the numerical values of

the value function (net present value). Note that only for a certain range of x_2 the net present value turns out to be positive. For the number of firms where x_2 is too small or too large, due to losses by firms with the new technology, the value of the value function can be negative. As our model is set up, where firms can be "locked in" by the new technology, for certain coordinates x_1, x_2 the value of (9) indeed can be negative, indicating debt as a discounted stream of deficits of firms. This, in the extreme, also permits a negative value function for a steady state [32] of the system (7)-(8). (This financial aspect may be interesting to pursue in future research).

Next let us consider the trajectory, resulting from (10), representing the (optimally controlled) market shares of the new and the old technologies. This is depicted in figure 6.

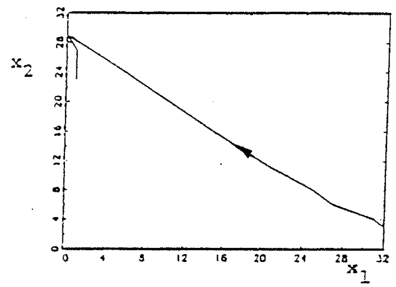

Figure 6. Optimal trajectories $_1$, x_2 $(r = 2)$.

Our dynamic optimization version uses the same parameter values as employed for the trajectory in figure 3; except the discount rate is now finite. As figure 6 reveals, starting with roughly the same initial conditions as in Figure 3 the trajectory of our dynamic optimization problem exhibits nearly the same features as the previous graph. Moreover, the trajectory approaches an equilibrium close to (E1) of figure 3. Note that such results can hold only approximately, since in the dynamic optimization problem (7)-(8) r also enters as parameter of the trajectories and the equilibria.

We have replicated only one model specification here, but other trajectories, discussed in the previous section for the case $r \to \infty$, should also reemerge in the present case for finite but large discount rates r. In intermediate ranges of the discount rate r, however – where r may also act as a bifurcation parameter – there might be different behaviors of the optimal trajectories with variations in r.

IV. CONCLUSIONS

Our proposed model of innovation and diffusion posits that knowledge of new technologies is obtained only by inventive investment on which a return is expected. In this sense it shares the feature of imperfect information models where information is only obtained at a certain cost. When, however, the new technology is available it has to compete with existing technologies. In contrast to standard models of diffusion processes which utilize the logistic approach with a unique number of dominating firms (the innovators) our proposed model operates with two groups of competing firms. As demonstrated, more general interaction effects among the two groups of firms or competing technologies (predator-prey, learning or spillover, and crowding effects) appear to be desirable ingredients in the modeling of the dynamics of technical change.

Because of the quite intricate interaction of the different types of firms, trajectories exhibit considerable diversity. We particularly studied the impact of the variation of diffusion speed, and the alteration of initial conditions on the trajectories. As shown innovating firms that undertake an optimal inventive investment may succeed, but firms with the old technology may still coexist side by side with the new technology. Innovating firms may also dissipate for a certain diffusion speed or initial conditions. Thus, high but also low market shares can emerge for the innovating firms.

Other results from studies of technological change with one variable – one technique only – do not necessarily carry over to models with two variables. In Arthur (1988, 1989), for example, a technology with positive feedbacks, due to increasing returns to adoption, always dominates the innovation process. With more than one variable involved in the diffusion process, characteristic for our model in sections II and III, a different distinctive pattern of the innovation process can arise. In the above model a positive feedback mechanism for the new technology may indeed dominate the dynamics for a while. Yet, at a later period, due to the interaction of competing techniques (resulting from crowding effects for the new technology and learning effects for the old firms), the innovating firms' market share may again decline [33].

Appendix: Numerical Specifications of the Model

1. For the simulations of the model with optimizing behavior of firms in sections II and III the following parameters were chosen $\phi = 5$; $c = 0.1$; $c_0 = 0.04$; $k = 1$; $v = 0.5$; $\gamma = 0.8$; $e = 0.01$; $\alpha = 1$; $\beta = 0.2$.

The parameter a took the values 0.01, 0.02, 0.03, 0.04. For $a = 0.01$ and $a = 0.02$ there exist four equilibria. For $a = 0.02$ the equilibria are given by

$$(E1): x_1^* = 0.6,\ x_2^* = 17.6; \qquad (E2): x_1^* = 20,\ x_2^* = 0;$$
$$(E3): x_1^* = 26.7,\ x_2^* = 0.3; \qquad (E4): x_1^* = 18.1,\ x_2^* = 0.6;$$

For $a = 0.03$ and $a = 0.04$ there exist two equilibria. For $a = 0.03$ the equilibria are

$$(E1): x_1^* = 0.4,\ x_2^* = 17.8; \qquad (E_2): x_1^* = 20,\ x_2^* = 0;$$

For $a = 0.04$ the equilibria are given by

$$(E1): x_1^* = 0.3,\ x_2^* = 17.9; \qquad (E_2): x_1^* = 20,\ x_2^* = 0;$$

2. For the above parameters it can numerically be shown that Propositions 1-3 of section II hold. The optimal u is:

$$u^* = (\sqrt{\phi x_2/c} - \phi)/x_2.$$

For g we obtain

$$g = u^* x_2/(\phi + x_2 u^*) - cu^* - c_0 x_2$$

For the parameters chosen we obtain for $g(\cdot) = 0$ a $x_{2\,\min} = 0.73$ (our lowest number of innovating firms to allow for nonnegative profits). On the other hand x_2 should be smaller than $x_{2\,\max}$ which for our numerical values is roughly 17. Thus, any $x_{2\,\min} < x_2 < x_{2\,\max}$ will generate for an optimal u^* a $g(\cdot) > 0$. For $x_{2\,\max} = 17$ we get again $g(\cdot) = 0$. Greater x_2 will generate losses even for the optimal control u^*. This numerically demonstrates the Propositions 1-3. For the given parameters the critical value at which $\partial g(\cdot)/\partial x_2$ starts declining is $x_2 = 7$.

Note, however, that firms of type x_2 will still optimize even if losses occur, since optimizing in the presence of fixed costs minimizes losses. For example, let the minimum number of firms $x_{2\,\min}$ be 0.73 at which $g = 0$. Here we have $u^* = 1.4$. If firms choose an activity level where $u = 2$ we get a $g = -0.003$ and for a $u = 1$ we have a $g = -0.001$. For $x_2 = x_{2\,\min}$ the optimal u^* gives $g(\cdot) = 0$. For $x_2 < x_{2\,\min}$ or $x_2 > x_{2\,\max}$ the optimal u^* minimizes losses (when firms are locked in). On the other hand a $x_2 = 0.5$ results in $u^* = 0$ and thus the inventive investment in new technology would halt (though there is still a fixed cost to be paid).

Notes and References

1. The new information theory, for example, elaborated by Arrow (1962, 1987), views innovation as a costly "production of information".

2. In the "New growth Theory", as for example put forward by Lucas (1988) and Romer (1990*b*), a central issue has become "to endogenize innovation". This research effort aims at a "rigorous accounting of resources used up in creating new knowledge" (Grossman and Helpman, 1990, p. 87).

3. Below we will not apply the competitive condition that for the innovation the technological rent is instantly dissipating so that the cost of innovation is always equal the present value of profit flows; an assumption made, for example, by Romer (1990*b*) and Young (1992). We will admit positive and negative net present values.

4. *Cf.* for example, Dasgupta and Stiglitz (1980*a*, 1980*b*).

5. For a related view, *cf.* David (1990).

6. For the evolutionary approach *cf.* Nelson and Winter (1982), Winter (1984), Prigogine (1976), Silverberg (1982), Silverberg *et al.* (1988), and Bluemle (1989). For a related model that works with methods of mathematical biology, *cf.* Flaschel and Semmler (1987, 1991).

7. Less uniformity and instead diversity with respect to behavior and techniques would be predicted to empirically exist (*cf.* Silverberg *et al.*, 1988; Winter, 1984; and for an interesting complete specification of such an approach, *cf.* Conlisk, 1989).

8. Risk and uncertainty will be referred to in connection with the discount rate to be discussed later. A more fully developed stochastic version is proposed elsewhere, *cf.* Semmler (1994).

9. Our model lends itself to a broader interpretation. The classification of active versus passive behavior appears to be relevant for the process of innovation and diffusion (i) within industries (innovating versus imitating firms), (ii) between industries (technological leading versus adapting sectors), (iii) between size classes of firms and (iv) between regions and countries (leading versus adapting countries).

10. This is in contrast to Romer (1990*b*) and Young (1992) who assume immediate rent dissipation for the innovating firms.

11. Smith (1968) and Berck (1979) have applied such an idea first to the extraction of natural resources.

12. Interaction effects of similar type were first discussed in sales-advertising models, *cf.* Dodson and Muller (1978) and Feichtinger (1992).

13. For a use of an inverse demand function in the context of an innovation model, *cf.* Dasgupta and Stiglitz (1980*a*, 1980*b*).

14. In our model it will be thus only for an initial stage that there are positive feedbacks, self-enforcing mechanisms, or processes of cumulative causation working. Due to the different types of interactions positive feedbacks may be turned into negative ones.

15. In Sieveking and Semmler (1996) it is shown that maximizing a value function for a discount rate r approaching infinity the trajectories x_r approach the trajectories x of a system with a time horizon approaching zero.

16. We posit that through the effort spent there is an amount of "technological knowledge" ("information" in the sense of Arrow, 1962, or "stock of design" as in Romer, 1990*b*) obtained. The "technological knowledge" can be thought of being

created by (i) hiring engineers, (ii) running research laboratories or (iii) purchasing information on new technologies. In principle, however, the effort spent might not always lead to more technological knowledge and successful implementation of a new technology. There is considerable uncertainty and risk involved (*cf.* Arrow, 1962). A stochastic version of the above model can be formulated along the lines of Dasgupta and Stiglitz (1980*a*). We could write hu where the variable h represents a stochastic variable. It denotes the "probability of success" for a unit of effort spent. We have kept $h = 1$; *cf.* Semmler (1994).

17. We assume here that the new technology (new knowledge) is used jointly by all firms x_2. When the technology is operated by the firms the output resulting from it is posited to be proportional to the number of firms. Technological knowledge has the character of a public good for the cooperating firms; *cf.* also Romer (1990*a*).

18. Note that we have chosen here a formulation of the (net) price (mark-up) where the price is proportional to cost. A more complicated cost function (with first decreasing then increasing cost) could easily be included in the return function; *cf.* Dasgupta and Stiglitz (1980*a*, 1980*b*).

19. Iwai (1984) alternatively assumes that the innovating firms inside the industry increase their growth potentials and thus grow faster through $vg(\cdot)$. Our specification above leaves open this interpretation.

20. Our specific formulation of the predator-prey interaction can be justified in more general terms without reference to the sales-advertising model. A general form of a predator-prey interaction can be: $a(x_2)x_1$ with $a(x_2) = ax_2 x_2$. The function $a(\cdot)$ is called a "trophic function". There are many forms of such a function discussed in the literature, *cf.* Svirezhev and Logofet (1983).

21. In our context, learning by old firms means that they improve their production processes when information about the new technology spreads and the competitive pressure from the new technology increases. It is reasonable to posit that information about the new technology leaks out faster the larger the number of firms that apply the new technology. Our assumption means that the diffusion speed accelerates with x_2.

22. Note that for the return function and (3), (4) u^* is required to be nonnegative so that with x_2 small u^* becomes zero.

23. In general, the subsequently stated concavity of our return function does not necessarily hold. For $g(\cdot) \geq 0$ or even for $u^* \geq$ points of inflection migth occur. For our numerical parameters, however, the statement is correct.

24. The concavity holds for all $u^* \geq 0$. For $u^* = 0$ which is admitted we get $g(\cdot) < 0$. Yet, for out parameters concavity still holds.

25. The other equilibria which result for different values of a and the rest of the parameters are reported in appendix 1. These computations were made with the software package "Mathematica" (*see* Wolfram Research, 1991).

26. Empirical studies on the diffusion speed of new products among countries suggest that the diffusion parameter is around 0.02 (*cf.* Jovanovic and Lach, 1990). We start with a diffusion parameter $a = 0.01$ which is then subsequently increased. It is worth noting that we did not experience an important role of ϕ as bifurcation parameter.

27. Because of limited space the variety of graphs depicting the change of the local dynamics at (E1), (E2), (E4) – resulting from a change of the diffusion parameter a or initial conditions – are not included here. They are available from the authors upon request.

28. The local dynamics of (E2) can also analytically be computed since there $J_{21} = 0$ and, due to $\partial vg(\cdot)/\partial x_2 > 0$ for small x_2, we possibly get $J_{22} > 0$ and therefore $Tr\,J > 0$ (or < 0). In the first case (E2) would be a repeller; in the second case an attractor.

29. An analytical method to study problems of dynamics with multiple equilibria, some of them with saddle point properties, is provided in Sieveking and Semmler (1996).

30. For details of the dynamic programming algorithm applied, *cf.* Semmler (1995).

31. For the relation of risk and the discount rate, *cf.* Reed (1984).

32. Of course, if firms are sufficiently mobile, with a negative net present value they would tend to exit that activity in the long run.

33. In addition, as the value function shows, it is possible that there might be certain equilibria for the innovating firms as attractors which lead to the accumulation of debt by innovating firms. In the context of models with returns to scale in adoption the finance problem is particularly important. Since in the case of returns to scale firms are required to have a large market share to operate profitably, they may have to run temporary deficits and to borrow against the future before they dominate the market. This may turn out to be a quite risky operation. The financial aspect of innovation and diffusion processes appears to be an interesting topic for future studies; *cf.* Foley and Lazonick, 1990.

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LEARNING, SELECTION, AND EVOLUTIONARY PROCESS. SOME EXPLORATIONS IN FREQUENCY-DEPENDENCY SYSTEMS

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Abstract

In this paper we are attempting to describe how learning (private or public) change the process of competition selection among competing firms. By this way we introduce more heterogeneity in industrial structure. We basically use frequency dependency approach and replicator dynamics models.

Résumé

Dans cet article nous essayons de montrer comment l'apprentissage (privé ou public) modifie le processus de compétition sélection entre les firmes en présence. De ce fait nous introduisons une plus grande hétérogénéité dans la structure industrielle. Fondamentalement nous utilisons une approche de type « frequency-dependency » et des modèles de dynamique de réplication.

INTRODUCTION

Generally most analyses illustrate the main features of the evolutionary competition of an industry in a framework within firms differing in only one dimension: technology and internal organization (Metcalfe, 1986, 1992). *We want here to extend this perspective by taking into account two aspects, differences in technology and differences in learning capabilities.* For this reason we shall assume that unit costs in each firm vary during the process of evolutionary selection. For us, selection means competition between firms

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